A DISCUSSION OF THE VARIOUS METHODS OF PRECISELY POSITIONING A SHIP FOR THE PURPOSE OF HYDROGRAPHIC SURVEYING

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A DISCUSSION OF THE VARIOUS METHODS OF PRECISELY POSITIONING A SHIP FOR THE PURPOSES OF HYDROGRAPHIC SURVEYING

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TABLE OF CONTENTS

Title	Page
ACKNOWLEDGEMENTS	iii
LIST OF ILLUSTRATIONS	v
1. Introduction	1
2. Discussion of Errors	7
3. Celestial Navigation	21
4. Sextant Three-Point Positioning	49
5. General Discussion of Electronic Positioning Systems.	61
6. Specific Electronic Positioning Systems	88
A. Shoran	88
B. EPI	93
C. Loran	95
D. Decca	103
E. Lorac	109
F. Raydist	117
CONCLUSIONS AND RECOMMENDATIONS	137
BIBLIOGRAPHY	142

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LIST OF ILLUSTRATIONS

Figure	Title	Page
1	Typical Error Curve	14
2	Error Circle	14
3	Error Ellipse	15
4	Error Ellipse When Lines of Position Do Not Intersect at 90°	15
5a	Celestial Sphere	19
5b	Celestial Triangle	20
6	Relationship of Observed and Computed Altitudes . .	22
7	Plotting Celestial Lines of Position	23
8	Derivation of Formula for Dip	29
9	Effect of Tilting a Marine Sextant	36
10	Basic Three-Point Resection Problem	55
11	Derivation of Three-Point Solution	55
12	Relationship of Wave Length, Cycle, and Frequency	64
13	Frequency Bands of Basic Spectrum	64
14	Maximum Range of Operation of a Ground Wave System Without Serious Sky Wave Interference	65
15	Minimum Errors of Fix	67
16	Relative Errors of Fix of Circular Systems	69
17	Principle of Circular Systems	70
18	Ground and Sky Waves	72
19	Principle of Hyperbolic Systems	73
20	Relative Errors of Fix of Hyperbolic Systems . . .	82
21	Relation of Position and Accuracy With Hyperbolic Systems	83

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Figure	Title	Page
22	Shoran System	87
23	Circular System Coverage for Angle Intersections of 30° to 150°	91
24	Derivation of Loran Equation	99
25	Loran Scope Presentation	100
26	Decca System	107
27	Lorac Basic System	111
28	Lorac Type A System	114
29	Raydist System	118

LIST OF TABLES

Table	Title	Page
I	Comparison of Short Methods of Celestial Navigation	42
II	Accuracy of Celestial Observations at Sea	47
III	Comparison of Positioning Systems	122

INTRODUCTION

The progress of any science goes hand in hand with the development of new instruments, methods, and techniques. No advances at all can be made if we content ourselves with old methods or instruments which, although satisfactory for the standards of the age in which they developed, have not kept up with advances in other sciences. The history of positioning at sea, as will be shown, attests to the truth of this maxim.

The purpose of this report is to discuss the various methods of precisely positioning a ship at sea for use in hydrographic surveying and to evaluate each in an attempt to select that system which will give optimum results for the conditions prevailing in a particular operating area. In hydrographic operations it is necessary by some means to determine the position at sea of the hydrographic survey vessel at the moment a measurement of the depth of water is obtained by echo sounder. A precise determination is usually taken every 3 minutes and the distance between fixes interpolated on the basis of time and speed. The requirement for the precision of the position determination is particularly critical in coastal areas, in operations involving the laying of trans-oceanic cable or for location of submerged objects. Since this involves a discussion of modern navigation techniques it is best to start with a brief review of the history and progress of navigation up to the present time.

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The first fundamental navigational problem to be solved was that of direction. Very early in his history man discovered that directions at sea could be determined easily by reference to the stars, especially Polaris, and by reference to the sun when it rises, sets, or is at its greatest altitude. But this was of no use with overcast skies. It was not until 1100 A. D. that the magnetic compass was developed for use at sea, permitting constant determination of direction regardless of weather.

The solution of latitude followed closely behind that of direction. Latitude is relatively easy to determine from the altitude of Polaris or from the greatest daily altitude of the sun. The only practical method of sailing up until the end of the 18th century was to sail with the wind to the latitude of the destination and then to tack across along the latitude circle until the opposite shore was reached. Since the instruments of that time were only accurate to one-half degree one can readily see that there was much beating up and down the coast to be done in order to find the final destination.

The last fundamental problem of navigation, the determination of longitude, was the most difficult to solve and many ingenious approaches to the problem had been proposed:

One idea was that the variation of the magnetic meridian varied with longitude. Sebastian Cabot on his death-bed announced that he had solved the riddle of longitude and it is thought that his solution depended on magnetic variation. However, investiga-

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tion of the earth's magnetic field revealed that there were "variations of the variation" caused by secular and periodic changes of the earth's field and that the magnetic approach to the longitude problem was not practical.

Jean Dominique Cassini in 1673 suggested that longitude might be determined by observations of the eclipses of Jupiter's satellites. This method required a telescope 19 feet long in order to obtain sufficient accuracy. Although useful and practical ashore, the carrying and mounting of such a large instrument aboard ship was not feasible.

Another proposal by the noted astronomers, Kepler and Halley, suggested obtaining longitude by means of moon occultations and eclipses. This plan was theoretically possible but the motions of the sun and moon were not sufficiently understood at that time to give an accuracy closer than two degrees of longitude. Further, occultations and eclipses did not take place over a wide enough area and occurred too infrequently to be useful at sea.

In 1514 Werner proposed measuring the angular distances between the moon and stars in order to obtain longitude. At first there were not enough data on the moon's motion to permit use of this method but by the beginning of the 18th century sufficient data were available to permit use of angular distances. It was a long and tedious computing job to obtain longitude by this method but it was the only practical method until the advent of the chronometer.

For many years scientists were aware that difference of longitude could be easily obtained by comparing the time of transit of a star over the observer's meridian with the time of transit over a standard meridian. Clocks of many types had been known for years but there was no clock available that would be sufficiently accurate under the various ship motions and changes in atmospheric conditions encountered at sea. The problem seemed insurmountable until John Harrison, in an attempt to gain a large cash prize offered for solution of the longitude problem, invented the first successful chronometer in 1734. This invention, which was of such vital importance to all navigators, was not fully tested and put into production until the latter half of the 18th century. With the arrival of the chronometer the three fundamental problems of navigation, determination of direction, latitude, and longitude, were finally solved.

There were additional developments in the technique of navigation but few changes in principles. These developments were the simultaneous but independent invention of the marine sextant in 1731, by Hadley in England and Godfrey in America, which for the first time permitted precise angular observation at sea:

The discovery by the American sea captain, Thomas Sumner in 1837 that a single observed altitude of a celestial body determined a small circle of position which for short distances could be treated as a straight line upon which the observer must lie;

The discovery in 1870 by the French Admiral Adolphe Laurent Anatole Marq de Blonde de Saint Hilaire that a Sumner line of position can be plotted from a knowledge of the computed azimuth and by comparing the observed altitude of a body with the altitude computed for an assumed position close to the true position. This is now known as the Marq St. Hilaire intercept method which is the basis of modern celestial navigation.

For the 70 years from the development of the Marq St. Hilaire method to the beginning of World War II with its rapid advances in electronics, navigators contented themselves with the old and tried methods of navigation. They were so set in their ways that marine navigators bitterly fought the introduction of tabular devices such as Greenwich hour angle and sidereal hour angle which the modern air navigator demanded in order to obtain a speedy solution.

The progress of precise determination of position for hydrographic surveys was equally slow. In 1800 Captain Huddart, a British mariner, invented the three-arm protractor for mechanical solution of the mathematically involved "three-point" re-section problem. This was practically the only precise method of positioning survey craft until the end of World War II, despite its limited range which seldom exceeds ten miles from the coast.

Modern electronic positioning devices, developed since World War II, have many advantages in accuracy, range, and especially reliability. Although subject to breakdown, this does not occur nearly as often as does bad weather, darkness, haze, refraction, or many of the other things that plague the visual

observer. Research is constantly being carried out to develop improved systems with greater range and accuracy so that within a few years we may have world wide coverage for accurate positioning anywhere on the oceans.

The present level of navigation is the result of years of astronomical research and observations, theoretical investigations of various mathematical methods, and development of a great number of instrumental aids. However, in order to keep up with the many rapid advances in aircraft design, increased propulsive power, and the development of guided missiles, the science of navigation will have to make a continued search for improved instruments, methods, and techniques for precise positioning.

DISCUSSION OF ERRORS

In the precise measurement of any quantity the question of accuracy of the measurement always arises. In order to obtain more accurate values than would be given by a single observation the measurements are usually repeated a number of times. Under these circumstances it will invariably be found that the different measurements give different results. The cause of the discrepancies between different observations is that every measurement is subject to error. These errors can be divided into two broad classes: systematic and random.

Systematic errors are those which are of a constant value, or whose presence and magnitude are due to some fixed cause, many of which can be eliminated by the proper calibration of the equipment. Systematic errors may be of several classes designated as follows:

THEORETICAL ERRORS such as those due to refraction; aberration of light; effects of atmospheric conditions; propagational errors in the velocity of light, radio, or sound waves; sky wave errors; etc.

INSTRUMENTAL ERRORS such as those due to errors of division of graduated scales; defects in micrometer or vernier screws; eccentricity of the arc of a sextant; transmitting and/or receiving errors inherent in electronic equipment.

GEOMETRICAL ERRORS such as those due to the change of diver-

gence of hyperbolic lines of position between the center line and the foci in hyperbolic electronic positioning systems. Geometrical errors vary with the observer's location in the area.

PERSONAL ERRORS are due to personal peculiarities of an observer. For instance, an observer may always estimate an angle too small or always answer a signal too soon. Each observer has his own personal error which is different from that of every other observer.

Random errors, also called accidental errors, are those that still remain after all known constant errors and all evident mistakes have been carefully investigated and eliminated from the results. They are characterized by the fact that there are supposed to be as many positive random errors as negative ones of the same magnitude, and that there are many more small errors than there are large ones. Examples of random errors are the changing refraction of the atmosphere; imperfections of the touch and sight of the observer which render it impossible for him to handle his instruments with sufficient delicacy, estimate small divisions of graduation, or keep the instrument in continual adjustment; careless reading of the instrument (not a mistake or blunder); misalignment of electronic pulses; failure to have exact tangency of a celestial body and the horizon when using the sextant; accidental variations of velocity of propagation of radio waves; and accidental frequency errors.

Although at first it might seem that such irregular and

unpredictable random errors would be most difficult to investigate mathematically it will be shown that they are governed by a very precise law, the law of probability, and follow a regular pattern of distribution. Adjustment of observations by the method of least squares can only be done by treating as random errors all errors remaining after correction for known systematic errors.

No technical paper would be complete without at least one integral formula so it will be proper at this point to introduce the equation for the probability curve. The probability that an error will be between any two limits x' and x may be expressed:

$$P = \frac{k}{dx'} \int_{x'}^{x} e^{-k^2 x^2} \, dx$$

The "probability of an error" is a function of that error; so by letting x equal the error and y its probability, the law of probability of error is represented by an equation:

$$y = f(x)$$

If y is taken as the ordinate and x as the abscissa the resulting curve must satisfy the following requirements:

1. The maximum ordinate OA must correspond to zero error.
2. As x increases numerically the value of y must decrease.
3. When x becomes very large y must approach zero.

A typical curve is shown in Figure 1. Since different measurements have different degrees of accuracy it must be remembered that each class of observations will have a similar but

distinct curve of its own. The error curve has been deduced by two methods. Hagen's method rests on the hypothesis that:

"An error is the sum of an indefinitely great number of small elementary errors which are all equal and each of which is equally likely to be positive or negative".

Gauss' method rests on the hypothesis that:

"The most probable value of a quantity which is observed directly several times with equal care is the arithetical mean of the measurements".

Both of these methods express the equation of the probability curve as,

$$y = k\, e^{-h^2 x^2}$$

where
y = probability
h = constant
e = abstract number

There are numerous standards used to describe the accuracy of observations, some of the more prominent of which are:

<u>MAXIMUM ERROR</u>: An error for which no greater error has been found. If a great number of observations have been made it is probable that no greater error will occur and that all other errors should be less than the maximum error.

<u>MEAN ERROR</u> (m): Also called the standard error, root mean square error, or square mean error. It is the error whose square is the mean of the squares of all the errors. It can be expressed by the equation.

$$m = \sqrt{\frac{(U_1 - M)^2 + (U_2 - M)^2 + \ldots\ (U_n - M)^2}{n-1}}$$

Where U_1 U_2 ... U_n are the individual observations, n is the number of observations, and M is the arithmetic mean which is

equal to $\frac{U_1 + U_2 + \dots U_n}{n}$

In Figure 1 the mean error occurs at the inflection point of the curve (where the curve changes from concave to convex).

PROBABLE ERROR (p.e.): Also called the median error. It is an error of such a value that any given error is as likely to exceed it as be less than it. In other words, there is a 50% chance that another error will be greater than the given error and a 50% chance that it will be less. In Figure 1 the area under the curve from the y axis to the line representing probable error is equal to the area under the curve and to the right of p.e. line. Its relationship to mean error can be expressed by

$$p.\ e. = .6745\ m$$

AVERAGE ERROR (t): Also called the average deviation (a. d.). It is the arithmetical mean of all the errors disregarding the signs of the errors. Its main disadvantage is that it cannot be easily used in theoretical formulas. It may be expressed by the formula:

$$t = a.\ d. = \frac{\text{sum of the errors}}{\text{number of observations}}$$

Its relationship to mean error is $t = 4/5\ m. = 0.8m$

HUGE ERROR (a): An error of such magnitude that 999 out of 1,000 errors are less than it and only 1 is as large or larger than it. Its relationship to mean error may be expressed by:

$$a = 3.3m$$

In determination of error, the mean error is used because it is the inflection point on the Gaussean error curve and can also be used to correlate several types of errors to get the effective total error. The probable error is most often used in America, but since it represents only 50% probability it is not of much use to the navigator. The navigator who is trying to maneuver past a mine field, is in the vicinity of a danger to navigation, or is trying to locate himself precisely in order to locate a submarine cable is not interested in low probabilities. He wants to be as close to 100% certain as possible that his position is in a certain area. Therefore, the following table has been prepared to provide a factor which may be multiplied by the mean error to determine the desired probability level (percentage error):

Error	Probable	Average Deviation	Mean	-	-	-	Huge
Symbol	p.e.	t. or a.d.	m	-	-	-	a
Probability level (% error)	50%	58%	67%	90%	93%	99%	99.9%
Factor	.6745	.8	1	1.5	2	2.58	3.3

For example: If a position has been determined to have a mean error of ± 100 yards there is a 67% probability that the observer is within a circle of 100 yards radius. There is a 50% probability that he is within a circle of 67.45 yards radius; a 90% probability that he will be within a circle of 150 yards radius; and a 99.9% chance he will be within a circle of 330 yards radius.

There are two important points in the above example which

need clarification in order to prevent any misconception about errors.

First: The true value of any observation (in this case the position of the observer) can <u>NEVER</u> be known for certain. The error is always based on the correctness of the observations that were used in computing the error. If the observations were misread, the wrong signal sighted on, or the star misidentified the value of the observation will be in error but the amount will never be known. For example, suppose a sinking ship with a million dollars worth of gold aboard) reports its position as Latitude 50°00'00" North, Longitude 60°00'00" West and also reports that its mean error of position if $\pm$ 100 yards. Now further suppose that there was either an undetected error in transmission or the navigator made a blunder in his computations so that the true position actually was Latitude 50°00'00" <u>SOUTH</u>, Longitude 60°00'00" West. Treasure seekers might go to the reported position and locate themselves with the most precise methods known but they would not recover a single penny because the TRUE position was in another hemisphere. The only thing that can be said about any series of observations is that, assuming the observations are correct and free from blunders or mistakes, there is a 67% probability that the mean error of the observation is M $\pm$ mean error.

Second: A position at sea is determined by the intersection of two lines of position. Since each line of position has a calculable error, the error of the point of intersection will de-

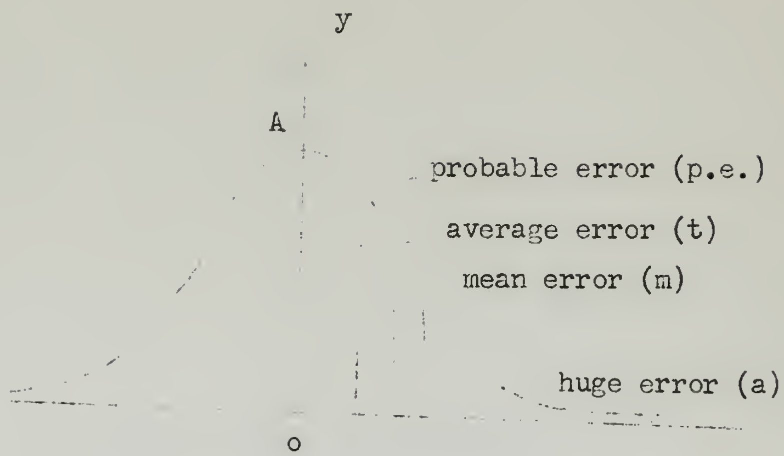
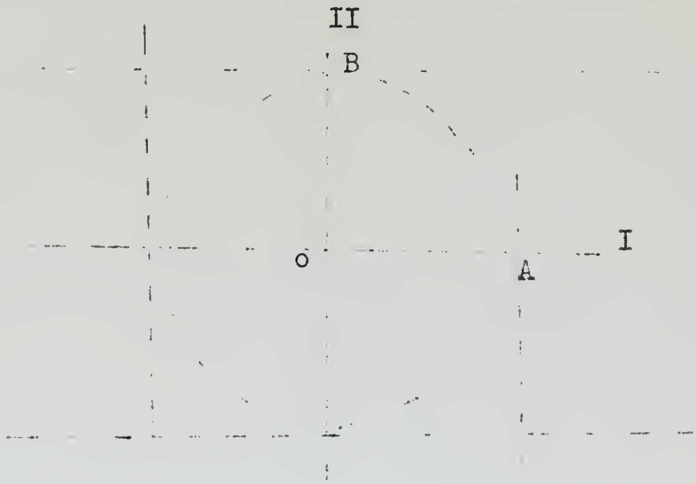
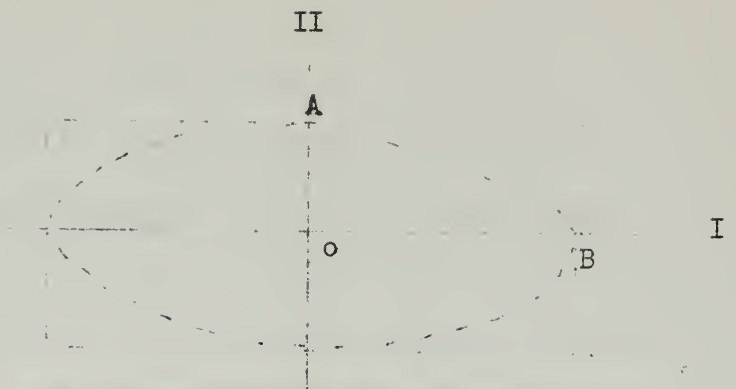


FIGURE 1: Typical error curve



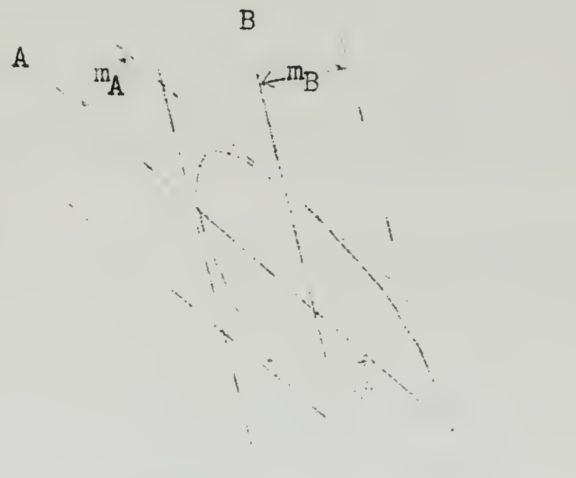
oA = oB = error in lines of position I and II

FIGURE 2: Error circle



oA = Error in line of position I
oB = Error in line of position II

FIGURE 3: Error ellipse



γ = Angle of intersection of lines of position A and B
m_A = Error in A
m_B = Error in B

FIGURE 4: Error ellipse when lines of position do not intersect at 90°

pend on the amount of the errors of the lines of position and also upon the angle of intersection of the lines of position. The error will describe an ellipse known as the standard error ellipse. If each line of position has an equal error and the lines intersect at right angles the error ellipse will be a circle (which is a special case of an ellipse where the major and minor semi axes are equal). If the errors are different but the lines of positions intersect at right angles, the major and minor semi axes of the standard error ellipse will be equal to the errors of the respective lines of position. If the errors are different and the lines of position do not intersect at right angles, the standard error ellipse will have major and minor semi axes dependent on the values of the errors of the lines of position and their angles of intersection. See Figures 2, 3, and 4.

In concluding this brief discussion on the theory of errors we should mention some of the formulas expressing how errors are propagated.

<u>Case I</u> Addition and/or subtraction of observations with known errors.

If A is an observation with error m_A

If B is an observation with error m_B

If C is an observation with error m_C

then the algebraic sum of these observations and its error may be expressed by the quantity ($D \pm m_D$).

Then $m_D = \sqrt{m_A^2 + m_B^2 + m_C^2}$

Example: Given: Longitude of Point A is $75^\circ 00' 00''$ West $\pm 2''.5$.

Point B is $10^\circ 15' 36'' \pm 3''.0$ West of Point A and Point C is $15^\circ 16' 30'' \pm 4''.0$ east of Point B.

Find: longitudes of Point B and Point C.

a) Longitude of Point B $= 75^\circ 00' 00''\ W + 10^\circ 15' 36'' \pm \sqrt{(2.5)^2 + (3.0)^2}$

$= 85^\circ 15' 36'' \pm \sqrt{15.25}$

$= 85^\circ 15' 36'' \pm 3.9''$

b) Longitude of Point C $= 75^\circ 00' 00'' + 10^\circ 15' 36'' - 15^\circ 16' 30'' \pm \sqrt{(2.5)^2 + (3.0)^2 + (4.0)^2}$

$= 69^\circ 59' 06'' \pm 5''.6$

or $= 85^\circ 15' 36''. - 15^\circ 16' 30'' \pm \sqrt{(3.9)^2 + (4.0)^2}$

$= 69^\circ 59' 12'' \pm 5''.6$

Both methods for getting longitude of Point C produce identical results which confirms the theory.

<u>Case II</u> Multiplication or division of observations with known errors.

Using same designation as in Case I let $(A \pm m_A) \cdot (B \pm m_B) = (C \pm m_C)$ (Multiplication)

then $$\frac{m_C}{C} = \sqrt{\left(\frac{m_A}{A}\right)^2 + \left(\frac{m_B}{B}\right)^2}$$

let $$\frac{A \pm m_A}{B \pm m_B} = (C \pm m_C)$$

then $$\frac{m_C}{C} = \sqrt{\left(\frac{m_A}{A}\right)^2 + \left(\frac{m_B}{B}\right)^2}$$

Example): If the sides of a rectangular field are measured and found to be

[illegible]

50 ft. $\pm$ 2 ft.

20 ft. $\pm$ 1 ft.

Find the area of field and its mean error.

$$(A \pm m_A)\ (B \pm m_B) = (C \pm m_C)$$

$$\text{Area} = 50 \times 20 = 1{,}000 \text{ sq. ft.}$$

$$m_C = 1{,}000 \sqrt{\left(\frac{2}{50}\right)^2 + \left(\frac{1}{20}\right)^2}$$

$$= 1{,}000 \sqrt{(.04)^2 + (.05)^2} = 1{,}000 \sqrt{.0041}$$

$$= 1{,}000 \cdot (\pm .064) = \pm 64 \text{ sq.ft.}$$

Therefore, the area is 1,000 sq.ft. $\pm$ 64 sq. ft.

Example 2): If the area of a rectangular field is 1,000 sq. ft. $\pm$ 64 sq. ft. and one side of the field is 50 ft. $\pm$ 2 ft. Find the length of the other side and its error.

$$\frac{A \pm m_A}{B \pm m_B} = (C \pm m_C)$$

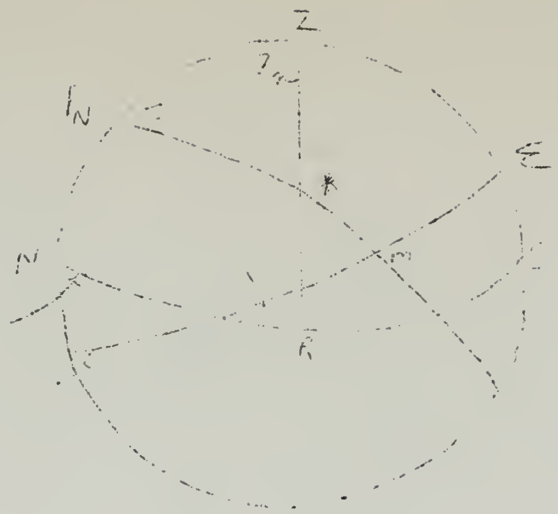
$$\text{Side} = \text{Area/Side} = \frac{1{,}000}{50} = 20 \text{ ft.}$$

$$m_C = 20 \sqrt{\left(\frac{64}{1000}\right)^2 + \left(\frac{2}{50}\right)^2} = 20 \sqrt{.0056}$$

$$m_C = \pm 1.4 \text{ ft.}$$

Therefore $C = 20$ ft. $\pm$ 1.4 ft.

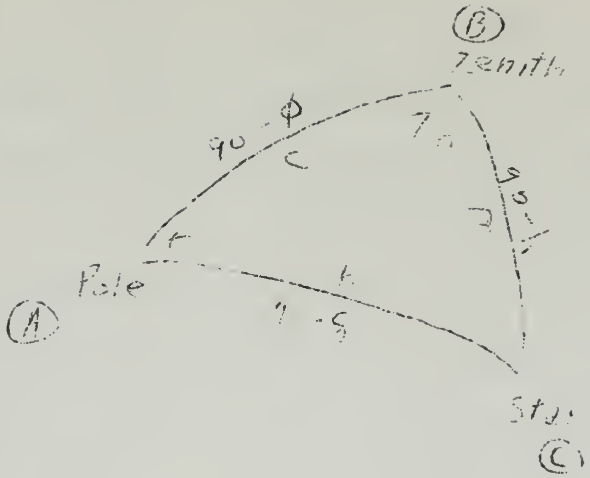
N. B. Since the true error is never known the sign is unknown. Therefore the square root of the sum of the squares of the mean errors is used to determine the resultant mean error. This resultant error is ALWAYS larger than any of the component errors. That is also the reason that the value of C determined in the Example 2 above does not check with the value of 20 ft. $\pm$ 1 ft. given in Example 1).



Explanation:

* is the star's position on celestial sphere.
Z is the observer's zenith.
P_n is the pole of earth's axis.
R* is the altitude of the star.
SR is geodetic azimuth measured from south point.
S is south point from observers position.
N is north point from observers position.
V is Vernal Equinox ( ).
E is intersection of celestial equator and meridian. It is always 90° from pole.
Em is hour angle = t.
Vm is right ascencion of star = x
m* is declination of star =
NP_nZES is observers meridian
Angle P_nZ* is azimuth from north = Zn
Pn* is polar distance of star = p
NP_n is latitude of observer
P_nZ is co.latitude of observer
Z* is zenith distance

FIGURE 5a: The celestial sphere



By substituting known values in appropriate spherical formulas, all unknown elements can be determined. Typical spherical triangle relationships are:

Sin law $\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$

cos law $\cos a = \cos b \cos c + \sin b \sin c \cos A$

sin cos law $\sin a \cos B = \sin c \cos b - \cos c \sin b \cos A$

FIGURE 5b: The celestial triangle (From Figure 5a)

3

CELESTIAL NAVIGATION

The oldest and most fundamental method of determining position at sea is by means of celestial observations. Figure 5 is a sketch of the celestial sphere illustrating the information and quantities required to solve for the observer's position.

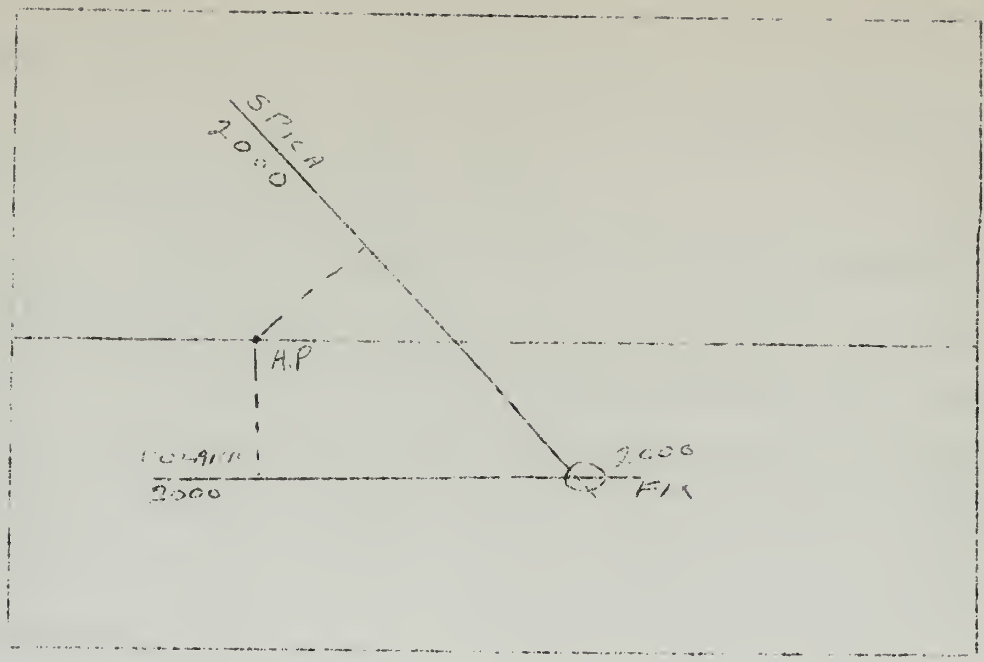
All of the modern methods employing celestial observations at sea depend on the altitude of two or more celestial bodies and recording the exact time each altitude is taken. The direct deduction of position on the surface of the earth from these observations involves such complex formulas and tedious computations that for practical purposes it is easier to use indirect methods. The Marq St. Hilaire intercept method based on the Sumner line of position is almost universally used today. In this method the navigator uses his dead reckoning position, or else uses an assumed position so as to reduce interpolation as much as possible. He then computes the altitude of the celestial bodies as viewed from the assumed positions. By comparing the observed altitude with the computed altitude and by using the computed azimuth a line of position can be plotted. This is done by drawing a line through the assumed position along the computed azimuth (or its reciprocal as necessary). The line of position will be perpendicular to the azimuth line at a distance from the assumed position depending on the difference between the observed and computed altitudes. As has been pointed out before, the line of position is actually the

o is the sub stellar point of star *.
A is the assumed position resulting in computed altitude h_c.
B is the ships actual position giving the observed altitude h_o.

If h_c is greater than h_o (as is the case in the sketch using B) then intercept AB is drawn in direction of the reciprocal of the computed azimuth. This is denoted A (away). See Figure 7.

If h_c is less than h_o (as is the case if ship is at point B') then intercept AB' is drawn in direction of computed azimuth. This is denoted by T (toward), see Figure 7.

FIGURE 6: Relationship of observed and computed altitudes



POLARIS		SPICA	
h_c	50° 05'.0	h_c	42° 36'.0
h_o	50° 00'.0	h_o	42 40'.0
	5'.0 A		4'.0 T

Computed Azimuth 000° Computed Azimuth 045°

At 2000 local time, simultaneous observations were made of two stars, Spica and Polaris, with results shown. For Polaris an intercept of 5.0 nautical miles (5'.0) is laid off along the RECIPROCAL of the computed azimuth and a line of position is constructed perpendicular to the intercept point. For Spica an intercept of 4.0 nautical miles is laid off in direction of the computed azimuth and a line of position constructed perpendicular to the intercept.

The intersection of the two lines of position is the position of the ship at 2000 local time.

FIGURE 7: Plotting lines of position

arc of a circle but for short distances can be treated as a straight line. The entire process can be understood best from a close inspection of Figures 6 and 7.

The line of position - intercept method described above can be handled in three ways:

1. By direct solution of the Pole-Zenith-Star astronomical triangle using four or five place trig tables. In addition to being a tedious process, formulas employing secants or tangents become very inaccurate for values near 90° and will not give acceptable results.

2. By splitting the astronomical triangle into two right triangles and solving by means of a combination of special tables and trig tables. In this mixed method the special tables involve triple interpolation for latitude, declination, and hour angle. This triple interpolation in some cases may introduce errors of 60' in first differences and the second or even third differences are far from neglibible. This system is seldom used.

3. By tabular methods (also called short methods) where computed altitude and azimuth can be obtained directly from special tables using as arguments latitude, declination and hour angle.

Basically the navigator is determining the position of a celestial body with respect to his own meridian, i.e. the body's local hour angle and declination. The required astronomical data is published in almanacs especially prepared for the navigator. Declination is tabulated in arc. Right ascension is tabulared for

use in any of the following formulas to obtain local hour angle:

$$LHA = LST - RA$$
$$LHA = GHA \pm \text{longitude } (+ \text{east}, - \text{west})$$
$$GHA* = GHA \text{ ARIES} - RA*$$
$$GHA* = GHA \text{ ARIES} + SHA*$$

where

LHA = local hour angle
GHA = Greenwich hour angle
LST = local sidereal time
RA = right ascension
SHA = sidereal hour angle. This is a meaningless angle used for ease of computation and equals 360° RA.

The errors that take place in positioning a vessel at sea may be broadly classified into the following categories:

Personal - due to the personal touch of each individual observer.

Instrumental - due to lack of proper instrument adjustment.

Observational - due to many sources of failure to properly align object and horizon.

Atmospheric - due to refraction and dip.

Computational - due to weaknesses or errors in formulas, tables, or methods used.

Plotting - due to distortions of chart paper, use of thick pencil lines, etc.

A list of the errors that occur in determining position by celestial means was published by Capt. P. V. H. Weems (13)[1] and is reproduced below. Arbitrary values have been assigned each error in order to give an idea of the size of the errors that may occur. As will be seen later, many of the errors listed below may sometimes have much larger values in actual practice. The

1 Numbers in parenthesis refer to the corresponding number in the bibliography.

[illegible]

purpose of the list is to show how the addition of a small error on a group of errors will not change the final results appreciably.

Error	Amount	(Amount)^2	Systematic (S) or Random (R)
Personal	0'.25	0'.0625	S
Refraction	0.05	0.0025	S
Dip	0.05	0.0025	S
Sextant	0.05	0.0025	S
Timing	0.05	0.0025	S
Defl. of Vert	0.05	0.0025	S
Horizon	0.05	0.0025	S
Chart	0.05	0.0025	S
Almanac	0.05	0.0025	R
Solution	0.04	0.0016	R
Plotting	0.02	0.0014	R
SUM	0.71	0.0845	

$$\text{Total error} = \sqrt{0.0845} = \pm 0'.29$$

PERSONAL ERROR (also called personal equation) of an observer is the tendency of a particular observer to consistently read angles too low, to respond to a signal too early, etc. It is extremely complex and difficult to analyse because it depends on the psychological make-up of the observer. It also varies with the position of the star in relation to the Pole as well as with the speed of motion of the star (this latter depends on the declination of the star). Professor C. H. Smiley, on the basis of 50,000

observations has determined that the personal error in using a sextant is at least $\pm 1'$. He states that any altitudes measured by a sextant and not corrected for personal equation must be regarded as uncertain by an amount which may be as large as $\pm 1'$. (10)

ATMOSPHERIC REFRACTION is the bending of a beam of light as it passes through the earth's atmosphere. In the navigational meaning of the term it includes both astronomical and terrestrial refractions since the beams of light from the star to the observer and from the horizon to the observer are both distorted by the effects of refraction.

Apart from the many other difficulties of observing, about 50% of the time there is at least $0'.4$ of unaccounted abnormal refraction. In addition there is no certainty that it is the same in all directions. Extra caution is needed where the horizon is fuzzy or shimmering, where a false horizon is suspected, or where sea temperature is expected to change radically within a few miles as when operating in the vicinity of the Gulf Stream. Due to radical changes in land temperatures, refraction is abnormal when within ten miles of land. After a rainstorm the refraction is frequently more than 15'.

Formulas for refraction are empirical for the most part but on the basis of theory it has been determined that refraction depends on air density and to a lesser extent on the exact composition of the air. Refraction tables are available for average conditions based on atmospheric pressure, height of observer, observed

altitude, humidity, and latitude. The exact correction is uncertain but Japanese tables indicate that there should be a 0'.1 correction for each degree of sea-air temperature difference. A ten degree temperature difference will mean 1' of refraction. The problem is very complex and does not show any promise of solution within the immediate future. It is complicated by such factors as mirage, atmospheric disturbances, and temperature differences between the height of eye of the observer and the surface. Further, there is no known method of measuring density and moisture content at the visible horizon which is usually many miles distant from the observer. Astronomers have been aware of the problem of refraction for many years and extensive research will continue until a solution is achieved.

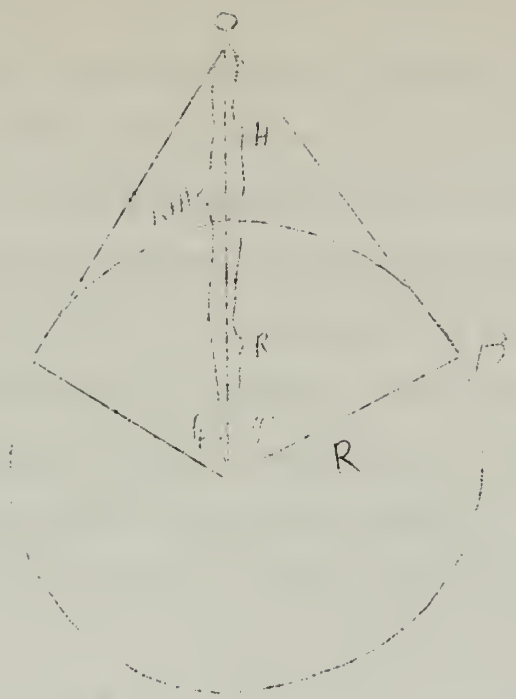
DIP is the vertical angle between the plane of the horizon and a line tangent to the apparent (visible) horizon. It can be seen from Figure 8 that dip (φ) and height of eye of the observer (H) are related by the formulas

$$\sec \varphi = \frac{R + H}{R}$$

$$H = R\,(\sec + 1)$$

where R is the radius of the earth.

It is also evident from plotting a graph of H against φ that the large changes in dip occur for small changes in H when H is less than 100 feet. The height of eye of an observer will change markedly in a rough sea due to roll and pitch of the ship



$$\cos\psi = \frac{R}{R + H}$$

Therefore $$\sec\psi = \frac{R + H}{R}$$

and $$H = R\,(\sec\psi - 1)$$

where H is height above sea level of observer at O
B is horizon of observer O
R is radius of earth
is dip of horizon

FIGURE 8: Derivation of formula for DIP

and this will produce a relatively large change in dip. This oscillation of the tangent to the horizon may amount to $2\frac{1}{2}'$ of arc in extreme cases, which corresponds to an error in position of $2\frac{1}{2}$ nautical miles. In order to reduce this error as much as possible it is necessary for the observer to know the height of eye for any place on the ship and for various conditions of loading. An accurate measurement should be made while the ship is tied to a pier and any subsequent change of draft noted accordingly. For most ships ten days steady steaming will result in a change of draft of about two feet which will affect the height of eye and also dip.

SEXTANT ERROR is the error caused by observational and instrumental errors occurring from use of the marine sextant. It is generally agreed that the accuracy of the marine sextant is on the order of $0'.1$ which corresponds to the smallest graduation on the micrometer drum. The accuracy of the sextant comes about through the ability of the human eye to detect a single break in a line, a break in a line of $0'.1$ of arc being readily detected by most observers. However, the theoretical accuracy attainable does not correspond with the actual accuracy attained. Observational errors will be discussed in the section dealing with horizon errors.

Instrumental sources of error in a marine sextant are: the resolving power, magnification, and curvature of the field of the telescope; backlash in the micrometer screw; variation in

the index error; possible distortion introduced by the glare and haze filters; and the eccentricity of the limb of the instrument.

Resolving power and magnification of a telescope should be selected so as to give the maximum field of view consistent with the minimum reading of the micrometer scale (0'.1). For instance, the average value of the resolving power of a telescope with 0.60 inch aperture is 0'.250 which is over twice the minimum micrometer graduation. Likewise, a 3X telescope will increase the resolving power of the human eye to 0'.15 which is still not as accurate as could be desired. However, increased magnification will result in a decrease in width of field which would be unacceptable. Sextant manufacturers have determined that a telescope of 3X magnification and 40 mm aperture will give the best width of field and smallest error due to difference between resolving power and smallest micrometer graduation.

Curvature in the lens of the telescope results in a lack of focus on the edges of the field of view. Any error due to such lack of focus can be avoided or greatly reduced by obtaining focus for the center of the field and always making coincidence or tangency in the center of the field.

Backlash is loose movement of the micrometer screw which may introduce errors of as much as 0'.2. The effects of backlash can be avoided by always turning the micrometer drum in a positive direction when making the final contact.

Index error is far from constant over the period of one

day's operation. Although index error can never be permanently removed because it is the residual error remaining after the instrument has been adjusted as much as possible it can be measured and its effect allowed for. It will vary from time to time with temperature changes, rough handling of the sextant, or loss of tension in the mirror screws, but its effects can be greatly reduced by noting the index error immediately before and after observations and correcting the observations accordingly. If the observations are to extend over a long period index errors should be noted at least every hour.

Ordinary telescope filters may introduce a small error due to uncertainties in the refractive index of the glass employed. However, most modern instruments are equipped with polarised filters which introduce no appreciable error.

Eccentricity of the limb is obtained by the manufacturer and a calibration curve supplied with each instrument to indicate the amount of correction necessary.

From the above it can be seen that a well adjusted sextant with proper observing techniques to reduce the effects of backlash and index error should be able to produce an accuracy equivalent to $\pm$ 0'.05 corresponding to a least graduation reading of 0'.1.

TIMING ERROR is the error introduced by faulty chronometers, inaccurate reading of the hack watch during observations, etc.At sea the usual accuracy with adequate time signals is 0.5

seconds with a decent hack. This corresponds to ± 0.25 seconds which compares favorably with the minimum reading of most comparing watches (0.4 seconds or $\pm$ 0.2 seconds). Time is of no great importance for routine celestial navigation because 4 seconds time error corresponds to 1' of arc. However, in precise work it is of tremendous importance. An accurate knowledge of time is necessary for the calibration and control of alternating current instruments, radio frequency control, etc. Obviously we will be unable to determine how many cycles per second are produced unless we know exactly how long a second is. But that is another subject.

DEFLECTION OF VERTICAL ERROR is caused by the gravitational attraction of submarine mounts or nearby shore formations which causes the direction of the vertical to change. Any change in the vertical will be equivalent to a change in the zenith, and this affects the altitude. The largest recorded deflection of the vertical on land is 1' in the vicinity of San Juan, Puerto Rico, but the average value on landis much less than 0'.5. There is as yet insufficient information on submarine topography to give exact values but there is evidence that deflections of the vertical will, in general, be less at sea. However, values of 1' at sea may be possible.

HORIZON ERROR is caused by the inability of the observer to properly make a celestial body tangent to the horizon. This may be due to movement of the horizon occurring when sights are taken from a ship in rough weather, and uncertainties due to a false or

hazy horizon. The effects of false and hazy horizons have been covered under the section dealing with atmospheric refraction.

During rough weather the horizon oscillates in proportion to the oscillating periods and is never constant even in the calmest waters. It is for this reason, incidentally, that a bubble octant, so useful for making aircraft observations independently of a horizon, cannot be used aboard ship. . . .
A bubble octant is calibrated in such a manner to average out periodic oscillations of the aircraft over a period of two minutes. The movements of a ship, however, are so erratic that it is impossible to center the bubble. For a marine sextant if the horizon is oscillating in tenths of a minute of arc it will be impossible to measure accurately the altitude of a celestial body within tenths of minute.

Another aspect of this same problem will be the error caused by failure to hold the sextant in a vertical position during observations in rough weather. It is standard practice to rock the sextant until the star image curves tangentially to the horizon. The touch between star and horizon can then be easily observed. The amount that the arc of the star fails at perfect tangency with the horizon will, of course, introduce an error into the observations. Cdr. J. Middendorp, R.N.N.R. was the first to discuss the different error that arises if the sextant is not held vertically, his original theory being subsequently slightly modified on a technical geometrical point by G. H. Clemence and Col. J. P. G.

Worlledge (18). In Figure 9 let

Z = observer's zenith

OS = axis of sextant

t = tilt of sextant in vertical

a = true altitude

Δa = error of altitude due to tilt

This produces the equation

$$\sin (a + \Delta a) = \sin a \sec t.$$

The error increases with increase in tilt and/or increase in stars altitude. Some values of this error for various angles of tilt and altitude are:

A tilt of 1° with 10° observed alt. gives an error 0'.1

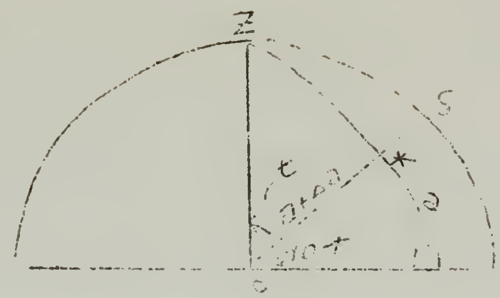
" " " 1° " 60° " " " " " 0'.9

" " " 4° " 60° " " " " " 14'.5

" " " 5° " 80° " " " " " 79'.5

<u>CHART ERROR</u> is the error that arises due to distortion of the chart on which the lines of position are plotted. The scale of the chart may be distorted up to 1% due to atmospheric changes in the chart house. Thus, the charting error depends on the scale of the plotting chart and the amount the chart has been distorted due to humidity, spilled coffee, etc. One can readily see that plotting on a scale of 1:500,000 or smaller (as are regulation plotting sheets) the error will not be negligible.

<u>ALMANAC ERROR</u> is the error due to inaccuracies in the almanac being used. Because the hour angle of the sun does not



Z is observers zenith
oS is axis of sextant
t is vertical tilt of sextant
a is true altitude

$$\frac{\sin a}{\sin 90-t} = \sin (a \pm \Delta a)$$

Therefore $\sin a \sec t = \sin (a \pm \Delta a)$

FIGURE 9: Effect of tilting a marine sextant

increase at the uniform rate of 30° per two hour interval as is assumed in interpolating for Greenwich Hour Angle (GHA) in the Nautical Almanac, an interpolation error of as much as 0'.7 is introduced in the value of GHA. The average interpolation error in GHA is 0'.14 for the entire year. This will introduce a proportionate error in the line of position depending on the method used in computing the line of position.

SOLUTION ERROR is the error caused by weaknesses in the method of computation or errors in the tables employed. Most tabular methods employ the formulas:

$$\sin h = \sin \phi \sin \delta + \cos \phi \cos \delta \cos H$$

$$\cos h \sin A = \cos \delta \sin H$$

$$\cos h \cos A = -\cos \phi \sin \delta + \sin \phi \cos \delta \cos H$$

setting $R = (\cos \delta \cos \alpha) \cos \Upsilon + (\cos \delta \sin \alpha) \sin \Upsilon$

and $S = (\cos \delta \cos \alpha) \sin \Upsilon - (\cos \delta \sin \alpha) \cos \Upsilon$

which substituting back in the original equations gives :

$\sin h = \sin \phi \sin \delta + R \cos \phi$ to give altitude

$\sin A = S \sec h$ to give azimuth

$\cos h \cos A = -\cos \phi \sin \delta + R \sin \phi$ to give the quadrant of the azimuth where

α = Right Ascension

δ = Declination

ϕ = Assumed latitude

h = altitude

A = azimuth

$H = \Upsilon - \alpha$

These formulas have none of the weaknesses of formulas using tangents and secants but there are other errors no less serious. For instance, the preparation of H. O. 249, one of the more popular short methods, involved the solution of 343,440 spherical triangles. One can easily imagine the amount of work involved in proofreading and the possibility of errors slipping by. There are also possibilities of interpolation errors due to the intervals involved for the various arguments. In computing by logarithms increased accuracy is obtained by the use of more decimal places but this is off-set by the greater time required to compute and by the greatly increased chances or error arising from the longer computations. The renowned Karl Friedrich Gauss claimed that the time required to compute 5,6, or 7 decimal places is in the ratio of 1 to 2 to 3. Using 5 place logs is therefore twice as fast as using increased places and the chances of errors are less because of the reduced amount of computation.

Table 1 is a comparison of the more popular short methods used in modern navigation as originally puolished in Navigation by Mr. Fred Franklin (17).

PLOTTING ERROR is the error arising from inaccuracies in the method of plotting, the errors of the plotting toold, and/or the method of arriving at the most probable position from the residual triangle. Due to the thickness of even the finest pencil line and to the errors in the plotting tools, the maximum precision possible for plotting is 1.0 yards for the best results and 4.0

[illegible]

[illegible]

[illegible]

yards for ordinary results when plotting on a scale of 1:10,000. These values are proportionately greater for smaller scales. A 1:50,000 harbor chart will have a plotting error between 5 and 20 yards; a 1:250,000 approach chart will have a plotting error between 25 and 100 yards; and a 1:1,000,000 standard plotting sheet will have an error between 100 and 400 yards.

An additional and perhaps greater source of plotting error arises from selecting the most probable position when three or more lines of position form a residual triangle (as usually happens in practice). Most navigators select the most probable position as being within the triangle: either in the center of the triangle or at the intersection of the angle bisectors of the triangle.

This assumption is correct if all the errors involved are random errors. However, if there is an unknown constant error in the observations which is larger than the random errors, then a situation may occur in which the most probable position is outside the triangle. The situation will arise, for instance, if all three stars are within an arc of 180° of azimuth so that the constant error is in the same direction for each star. Such a constant error may be caused by faulty index correction, by an erroneous estimation of height of eye of the observer, by an incorrect estimate of refraction, by the personal equation of the observer, or by unsuspected submarine gravity anomalies. Considering the possibility of the above errors occurring, the chances that the true

position WILL lie inside the triangle formed by three lines of position is only 1 in 4; that it will lie in the figure formed by four lines of position is 1 in 2. However, since there is little or no chance of correctly analyzing errors when at sea, the navigator is left with no choice but to continue his practice of selecting the true position inside the residual triangle. This short discussion should make him aware that too much confidence should not be placed in a position inside the triangle, especially when in the vicinity of hazardous waters.

The problem of the accuracy of astronomical observations at sea is of great interest to all who are in any way connected with the safe navigation of ships at sea. It is now being investigated by working committees of the American Institute of Navigation located at the University of California, Los Angeles, California and by the British Institute of Navigation located at the Royal Geographical Society, Kensington Core, London. Much work has already been done but under artificial circumstances such as observatiohs ashore, or in a vessel anchored in calm water, or by specially trained observers. The results obtained by such an approach may differ greatly from that obtained by a qualified navigator at sea where atmospheric conditions are uncertain, the state of the sea departs from absolute calmness, and where the navigator, although proficient in the technique of navigation, may be totally untrained in the theory of observations.

Table II is a summary of the results some of the investi-

gators obtained. As can be seen individual opinions vary widely. It can also be seen from the theory of the propagations of errors that due to the use of squared quantities the larger errors completely over-shadow the smaller ones and reduce them to virtual insignificance. It is evident that nothing will be gained by trying to reduce the small errors until the larger errors are removed. In celestial navigation the largest errors are those due to actual observing and it is these very observing errors that are least understood. Basically, a sight at sea is only as good as its observation no matter how carefully it is worked out.

Even if a number of observers at sea should get excellent agreement, such as was obtained by the U. S. Coast and Geodetic Survey observers in Table II, unknown factors, such as unsuspected gravity anomalies or errors in refraction, could introduce serious errors. For example a difference of 18° F. between water and air temperature will cause a refraction error amounting to 2 miles in the resultant line of position. As has been pointed out before, gravity anomalies on land may be as great as 1' and such an error might also occur due to unsuspected submarine anomalies.

On the basis of the data given in Table II it is probably safe to say that using the greatest possible care, specially trained observer teams, and precise observing equipment the best accuracy attainable is 0.4 nautical miles (mean error $\pm$ 0'.2). The huge error for use in dangerous waters will be $\pm$ 0'.66. For ordinary navigation the accuracy is probably a huge error of $\pm$ 5'.0. to give an accuracy of 10 miles.

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[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

TABLE I

COMPARISON OF SHORT METHODS OF CELESTIAL NAVIGATION

H. O. 211 (AGETON)

			ADD	SUBTRACT	ADD	SUBTRACT
GHA	37°32'.4	E.				
aLo	90°32'.4	E.				
t	53°	W.	A 9765			
d	16°22'.6	N.	B 1798	A 54987		
R			A 11563	B 19208	B 19208	A 11563
K	27°01'.5	N.		A 35779		
aLat	21°	S.				
K ~ L	47°01'.5				B 16642	
hc	25°58'.5				A 35850	B 4625
ho	26°35'.8				Z S121°32'W.	A 6938
a	37'.3 (Toward)				Zn 301°.5	

H. O. 208 (DREISONSTOK)

GHA	37°32'.4 W.					
aLo	90°32'.4 E.					
t	53° W.					
aLat	21° S.)	d 16°22'.6 N.				
t	53° W.)	b 57°28'.1 S.		A 17627	C 128	Z' 64°.6
		d 41°05'.5 S.		B 18219	D 59	
hc	25°59'		Sum	35846	Sum 187	Z" 57°.0
ho	26°35'.8					ZS 121°.6W.
a	36'.8 (Toward)					Zn 301°.6

(TABLE I)

(COMPARISON OF SHORT METHODS OF CELESTIAL NAVIGATION)

BRAGA'S METHOD

GHA 37°32'.4 E.

aLo 90°32'.4 E.

t 53° W.

aLat 20°37'.4 S.

d 16°22'.6 N.

L d 37°)	A 8064		
L d 4°14'.8)	B 19885	M - 121	
	B - 3	N 3	
	S 27946	P - 118	
			Z 58°.5
hc 26°10'.4			Zn301°.5
ho 26°35'.8			
A 25'.4 (Toward)			

AGETON'S 1942 METHOD

GHA 37°32'.4 E.

aLo 90°32'.4 E.

t 53° W.

		ADD	SUBTRACT	
aLat 21° S.)	d 16°22'.6N.			
t 53° W.)	K 32°31'.9S.	B 17627		Z' 64°.6
	K d 48°54'.5	B 18226	A 12282	
hc 25°58'.5		A 35853	B 4625	
ho 26°35'.8			A 7657	Z" 57°.0
a 37'.3 (Toward)				ZS121°.6W.
				Zn 301°.6

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

(TABLE I)

(COMPARISON OF SHORT METHODS OF CELESTIAL NAVIGATION)

WEEM'S LINE OF POSITION BOOK

GHA	37°32'.4 E.				
aLo	90°32'.4 E.				
t	53° W.				
aLat	21° S.)	d 16°22'.6 N.			
t	53° W.)	K 32°31'.9 S.		A	17627
		K d 48°54'.5		B	18219
hc	25°58'			Sum	35836
ho	26°36'		(From Diagram)	Z	N58°.4 W.
a	38' (Toward)			Zn	301°.6

WEEMS' NEW LINE OF POSITION TABLES

GHA	37°32'.4 E.						
aLo	90°32'.4 E.						
t	53° W.						
aLat	21° S.)	d 16°22'.6 N.					
t	53° W.)	K 32°31'.9 S.		A	17627	Z'	64°.6
		K d 48°54'.5		B	18219		
hc	25°59'			Sum	35846		
ho	26°35'.8		hc 26,	A	17627	Z"	57°.0
a	36'.8 (Toward)					Z	S121°.6W.
						Zn	301°.6

[illegible]

[illegible]

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[illegible]

[illegible]

[illegible]

(TABLE I)

(COMPARISON OF SHORT METHODS OF CELESTIAL NAVIGATION)

H. O. 214

GHA	37°32'.4 E.	aLat	21° S.
aLo	90°32'.4 E.	aDec	16°30'
t	53° W.	d diff	7'.4
Tab.hc	25°54'.5 d 56°	Z	S121°.6 W.
d x diff	4'.1	Zn	301°.6
hc	25°58'.6		
ho	26°35'.8		
a	37'.2 (Toward)		

H. O. 218

GHA	37°32'.4 E.				
aLo	90°32'.4 E.				
t	53° W.)	Tab. Hc	26°00'	t - 4	Z S122°W.
aLat	21° S.)	corr.	1		
ALDEBARAN	)	Hc	26°01'		
		Ho	26°37'.8		
		a	36'.8 (Toward)		Zn 302°

H. O. 249

GHA	284°17' W.		
aLo	90°43' E.	Correction 2' E. (For Plotting use 90°45' E.)	
	375°00'		
LHA	15°00')		
aLat	21° S.)	Hc 25°48'	
ALDEBARAN	)	Ho 26°37'.8	
		a 49'.8 (Toward)	Zn 301°

(TABLE I)

(COMPARISON OF SHORT METHODS OF CELESTIAL NAVIGATION)

LIEUWEN METHOD

GHA $37^\circ32'.4$ E.				
aLo $\underline{90^\circ32'.4\ \text{E.}}$				
t 53° W.				
aLat 21° S.)	K $32^\circ31'.9$ S.	A 17626	Z'	$64^\circ.6$
t 53° W.)	d $\underline{16^\circ22'.6\ \text{N.}}$			
	K d $48^\circ54'.5$	B $\underline{18226}$	Z"	$\underline{57^\circ.0}$
h $24^\circ58'.5$		C 35852	Z	$121^\circ.6$
c^1 $30'.0$				
c^2 $\underline{30'.0}$				
hc $25^\circ58'.5$				
ho $\underline{26^\circ35'.8}$				
a $37'.3$ (Toward)				

	H.O.211	H.O.208	Ageton's 1942 Method	Old Weems'	New Weems'	H.O.214
Book openings	7	4	4	3	4	2
Table entries	9	8	8	4	6	4
Additions and subtractions	7	6	6	4	5	4
Steps in diagram. . . .	—	—	—	6	—	--
TIME FACTOR	23	18	18	17	15	10
Number of pages	49	106	103	54	44	264*

	Braga's Method	H.O.218	H.O.249	Lieuwen Method
Book openings	3	2	1	5
Table entries	7	3	2	6
Additions and subtractions	4	3	1	4
TIME FACTOR	14	8	4	15
Number of pages	147	230*	322	160

* Per volume.

TABLE II

Results of Various Investigations in Determining the Accuracy of Celestial Observations at Sea.

A. Observations at Main Control Buoy "J" George Bank - Season of 1930 by USC and GS Ship Lydonia (13):

Observer	No. of Sets	Lat.	Long.
G. D. Cowie	8	41°29'.39	67°15'.63
L. S. Hubbard	4	41°29'.46	67°05'.53
W. M. Scaife	6	41°29'.60	67°15'.73

Probable error of the result in Lat. = ± 0'.052 = ± 312 ft.

Probable error of a single observation in Lat. = ± 0'.221 = ± 1,326 ft

Probable error of the result in Long. = ± 0'.061 = ± 366 ft.

Probable error of a single observation in Long. = ± 0'.258 = ± 1,548 ft.

The observations were made at sea from a moored ship by highly trained observers with more precise equipment than is available to the average navigator. The above results correspond to a mean error of ± 2,100 feet for a single observation and a huge error of ± 6,930 feet.

B. Observations of 1,500 altitudes of celestial bodies during a round trip sea voyage from U. S. A. to Brazil in 1947 by C. H. Smiley et al (10):

Probable error for single observation = ± 0'.27 = ± 1,670 ft.

Mean error for single observation = ± 0'.4 = ± 2,470 ft.

Huge error for single observation = ± 1'.32 = ± 8,151 ft.

(TABLE II)

(ACCURACY OF OBSERVATIONS AT SEA)

The observations were made from a moving ship by eight scientific observers well grounded in the theory of errors and theory of observations. The results were determined after allowance had been made for all sources of systematic error.

C. Observations taken in 1934 on board the FALSTERBOREV light vessel anchored five miles off the southwest coast of Sweden by B. Chr. Peterson (20).

525 altitudes grouped between 18°07' and 55°11' revealed a mean error of a single observation between ± 0'.58 and ± 1'.57. The variation probably was due to irregularity of the sea horizon and to abnormal terrestrial refraction.

365 sights divided into morning and afternoon observations revealed that both were about equally accurate:

No. of Sights	203 A.M.	162 P.M.
m	±1.'08	±1.'02

Correcting the above for sea motion, refraction, etc. gave an m = ±1.'5 to ± 2'.0 and a huge error a = ±4'.95 to ±6'.6.

These observations were taken from an anchored ship in calm water under excellent observing conditions. The results indicate an accuracy of 3 to 4 nautical miles for a mean error of ± 1'.5 to ± 2'.0 and an accuracy of 13.2 nautical miles for a huge error of ±6'.6.

SEXTANT THREE-POINT POSITIONING

In making hydrographic surveys the usual practice is to position the sounding vessel by simultaneously measuring with two sextants a pair of angles subtended by three known points, usually survey signals or other fixed objects ashore. Each position of the vessel as fixed by the two observed angles may theoretically be computed by solving the classical three-point resection problem, but this is such a horrendous task as to be impractical due to the long and elaborate computations involved.

In practice the procedure is for two observers to measure two angles simultaneously with sextants; one observer measures the angle between the left and center objects while the other observer measures the angle between the center and right objects. A graphical solution is then obtained by plotting with a three-arm protractor on which the two observed angles have been set. When the three arms of the protractor are made to coincide with the fixed objects as plotted on a chart, the position of the vessel at the time of the observation will be at the center of the protractor. A small opening is located at the center of the protractor through which a sharp pencil point or fine pricker may be inserted to mark the location of the "fix" on the chart. The advantages of this graphical method are that all operations are performed on board the survey vessel, the required data are known immediately, and the position may be determined quickly.

The theory of the three-point problem depends on the following principles:

1) A circle can be described through any three points.

2) If two of the points are fixed in position, the angle between them measured at an unknown third point located on the circle will be the same. Thus, a circle may be passed through the left and center fixed objects by using the observed angle subtending these objects. The position of the ship will be somewhere on the circumference of the circle.

3) If a circle is similarly passed through the center and right objects by use of the observed angle subtending them, then the position of the sounding craft is at the intersection of the two circles.

The strength of the three-point fix depends on the position of the ship in relation to the three fixed points. If the ship is on a circle passing through the three fixed points the solution will de indeterminate because the two circles will coincide and the protractor will swing along the arc of the coincident circles. This is called a "swinger". However, a strong fix is obtained when:

a) The observer is inside the triangle formed by the three fixed objects.

b) The sum of the two angles is equal to or greater than 180 degrees, and neither angle is less than 30 degrees.

c) The three objects are in a straight line.

d) The center object is closest to the observer.

e) At least one of the angles changes rapidly as the survey vessel moves from one location to another.

In addition, the sum of the angles should not be less than 50 degrees, better results being obtained when no angle is less than 30 degrees.

There are several serious disadvantages to the sextant three-point method which need to be considered. The method is useless during periods of decreased visibility or during periods of darkness. This means that in any area (except the Arctic where there may be long periods of daylight) the sextant method can only be used for 50% of the time, excluding additional interruptions due to weather and/or atmospheric conditions.

The range of operations is restricted to a very small distance from the coastline due to curvature of the earth and also to the inability of the human eye to detect objects which subtend an angle smaller than one minute of arc. This corresponds to viewing an object 1 inch in size at a distance of 300 feet. In survey operations visual acuity is further complicated by other factors such as the shape of the object; brightness characteristics; atmospheric conditions causing haze, mirage, glare, etc.; and the nature of the background color. Blue backgrounds (such as the sky) decrease by half the visual acuity obtainable from black or white backgrounds. The distance at which objects of various heights may be seen by persons with the formula

$$D = \frac{H}{.0002909\ A}$$

where D is the distance of the object in feet, H is the height of the object in feet, A is the visual size in minutes of visual angle subtended at the eye by the object. In this case A has a value of one minute of arc.

A signal must also have enough height to overcome the curvature of the earth and the effects of refraction. The elevation above sea level that a signal must have in order to be visible a certain distance may be determined from the formulas:

$$D = 1.15\sqrt{H}$$

or

$$H = 0.75\ D^2$$

where H is the height in feet of the signal above sea level, 1.15 is an approximate constant for curvature and refraction, and D is the distance in nautical miles of the signal from the horizon. The height of eye of a sounding boat is about 9 feet and of a survey ship either 25 feet or 50 feet depending on the size of the ship. Standard survey signals made of wood are 30 feet high, and steel towers are 100 feet high. From the above data and formulas we can determine the following:

a) A 30 foot tripod will subtend one minute of arc at 16.5 nautical miles; a 100 foot tower will subtend one minute at 55 nautical miles.

b) A height of eye of 9 feet has the horizon 3.45 miles away; 25 foot height of eye has the horizon 5.75

miles away; and a 50 foot height of eye has the horizon 8.1 miles away.

c) A 30 foot tripod will be at the limit of visibility for the required visual angle of one minute when the elevation of the terrain is 70 feet. Beyond this, any increase in elevation does not increase the range at which a 30 foot signal will be visible except with use of a telescope on the sextant. A 100 foot steel tower can be seen out to 55 nautical miles before subtending too small an angle, but this is obtained only if the elevation of the surrounding terrain is 1,600 feet.

d) Along a low coastline a 30 foot signal can be seen about 10 nautical miles by a sound boat, 14.4 nautical miles by a large survey vessel. The expensive 100 foot steel towers will be visible 15 nautical miles and 19.6 nautical miles by sound boats and survey ships respectively.

Since the above distances all refer to the furthest visible signal, it can readily be seen that, if no angle less than 30 degrees is observed, this method will permit sounding operations along a coastal strip only 10 miles wide.

In addition to the visibility and range restrictions, visual control by use of sextant angles requires a much denser concentration of main-scheme triangulation in order to locate the

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[illegible]

secondary signals required for sextant observations. Under ordinary conditions there should be a secondary shore signal every 3/4 mile to provide sufficient control for the sounding craft.

Mention should also be made of the correction that must be applied to the sextant angles if the signals are elevated much above sea level. If one signal is more elevated than the other a correction is applied in accordance with the following formula

$$\operatorname{Cos} V_c = \frac{\cos Vo}{\cos h}$$

where Vo is the observed or inclined angle, h is the angular elevation of the elevated object, and Vc is the corrected angle. If both objects are elevated sufficiently to require correction, the correct horizontal angle may be obtained from the formula

$$\cos Vc = \frac{\cos Vo - \sin h_1 \sin h_2}{\cos h_1 \cos h_2}$$

in which h_1 and h_2 are the angular elevations of the two objects. For ease in logarithmic computation this last formula may be rewritten

$$\cos \tfrac{1}{2} vc = \sqrt{\sec h_1 \sec h_2 \cos S \cos (S - Vo)}$$

where

$$S = \frac{Vo + h_1 + h_2}{2}$$

Due to the involved computations in solving the three-point problem very little work has been done in determining mathematically the error in position that results from a known error in the sextant reading. Mr. R. P. Bailey of the U. S. Navy Hydro-

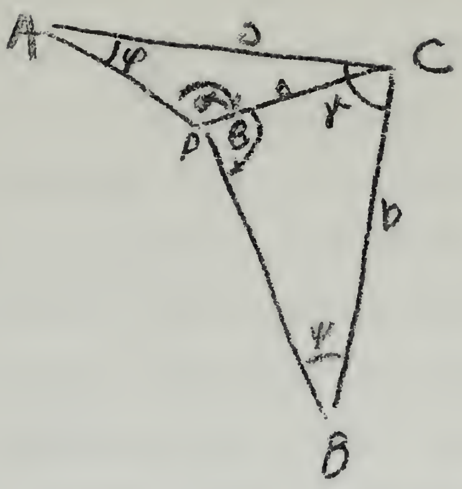
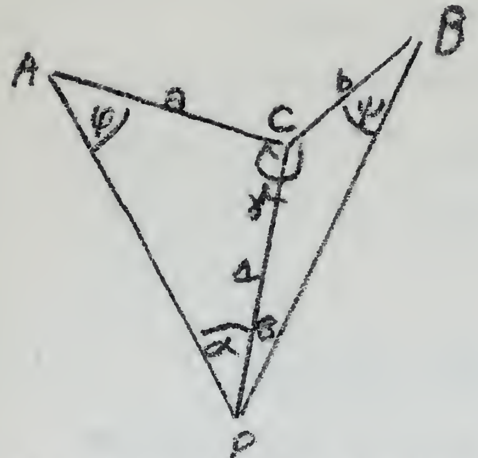


Fig 10 Basic Three-Point Resection Problem



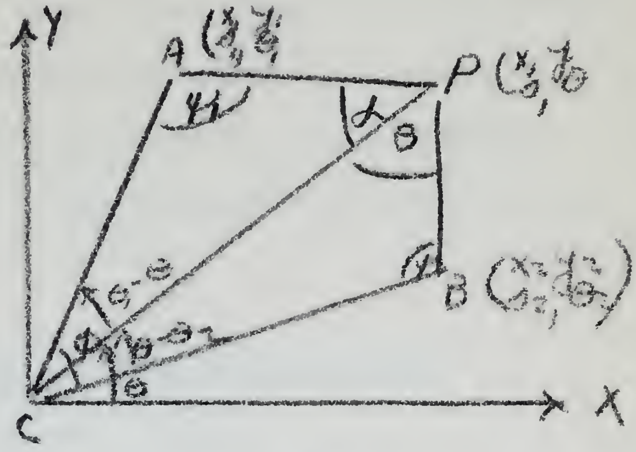
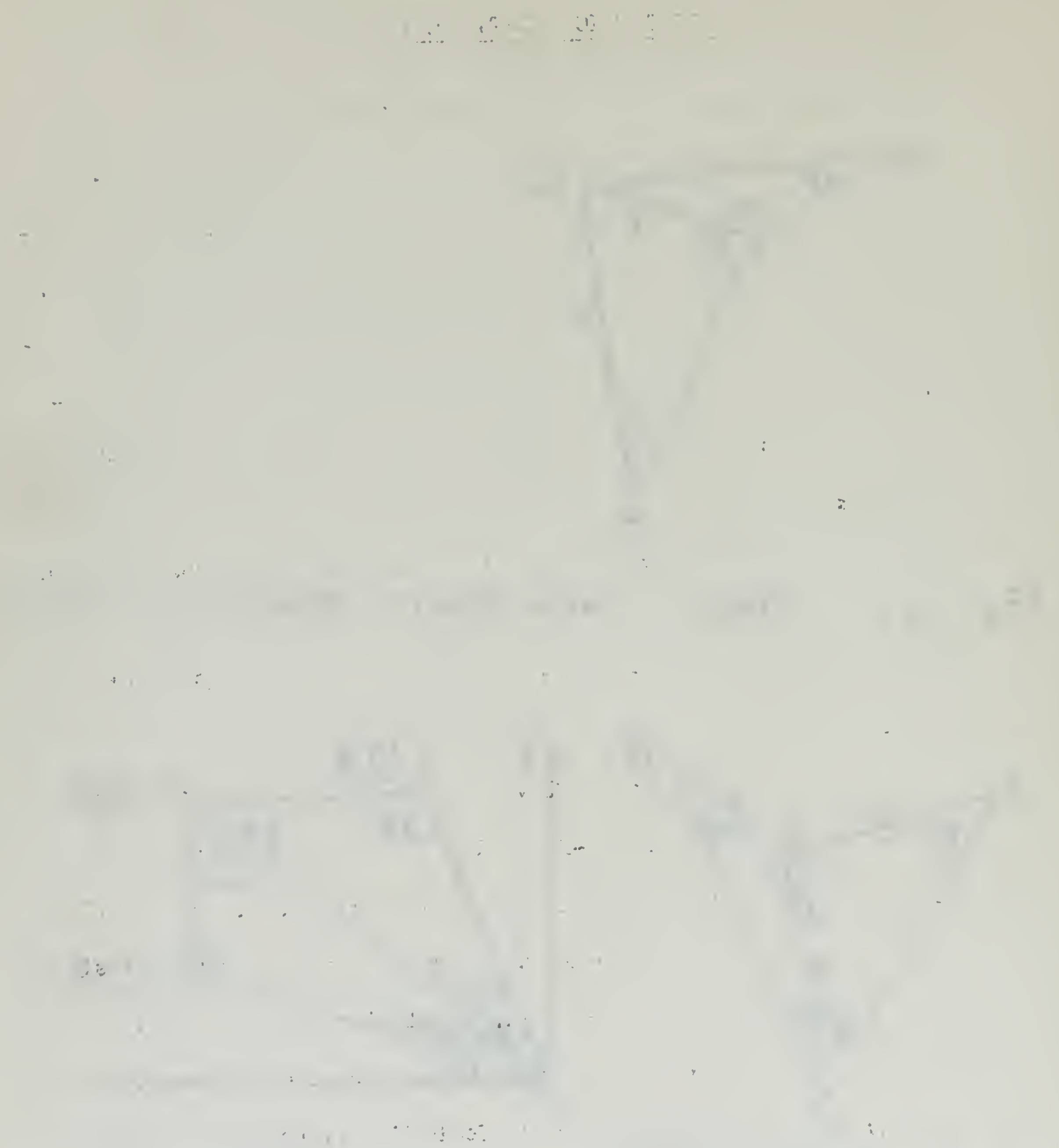


Fig 11 a) Burchhardt's Solution 11b Bailey's Solution



graphic Office has recently devised a very ingenious solution for the three-point problem and the error of position by use of polar coordinates but his results arrived too late for a detailed discussion in this report. The formulas derived by him will be included at the end of this chapter. In the absence of any mathematical method of determining the error of position, a graphical approach was used since a graphical solution to finding the actual position is used in practice. A position was carefully plotted on chart paper and then a $\pm$ 1 minute of arc change was applied to each angle. The resulting changes in position were carefully noted for different angular values. This crude method showed that sextant three-point fixes have an error of position of 10 yards on a scale of 1:10,000, being proportionately higher for smaller scales.

Mention has been made several times in this discussion of how involved the mathematical solution of the three-point problem can be. In support of these statements Burchhardt's derivation of the solution of the problem is as follows: (See Figure 11a).

Burchhardt's Solution of Three-Point Resection

Problem: coordinates of points A, B, C:

Known: X_A, X_B, X_C, Y_A, Y_B, Y_C;

Observed: Angles α and β .

Find: Coordinates of ship at point P.

The angle of intersectiion of the lines connecting the shore stations can be found by

$$\gamma = \text{Azimuth CA} - \text{Azimuth CB.}$$

Since (n-2) 180° is the number of degrees that the sum of the inside angles of an enclosed figure must equal:

$$\alpha + \beta + \gamma + \varphi + \psi = 360^\circ$$

Dividing by 2 and transposing we get

$$(1) \quad \tfrac{1}{2}(\varphi + \psi) = 180^\circ - \tfrac{1}{2}(\alpha + \beta + \gamma) = u$$

By the sin law

$$s = \frac{a \sin \varphi}{\sin \alpha} = \frac{b \sin \psi}{\sin \beta}$$

$$(2) \quad \frac{\sin \psi}{\sin \varphi} = \frac{a \sin \varphi}{b \sin \alpha} = \tan w$$

Subtracting $\frac{\sin \varphi}{\sin \varphi}$ from the left side and 1 from the right side, then multiplying each side by -1 we get

$$\frac{\sin \psi - \sin \varphi}{\sin \varphi} = \tan w - 1$$

$$(3) \quad \frac{\sin \varphi - \sin \psi}{\sin \varphi} = 1 - \tan w$$

Similarly, adding $\frac{\sin \varphi}{\sin \varphi}$ to the left side and 1 to the right side of formula 2 we get

$$(4) \quad \frac{\sin \varphi + \sin \psi}{\sin \varphi} = 1 + \tan w$$

Dividing formula 3 by formula 4:

$$\frac{\sin \varphi - \sin \psi}{\sin \varphi + \sin \psi} = \frac{1 - \tan w}{1 + \tan w} = \text{which can be rewritten:}$$

$$\frac{2 \sin \frac{1}{2}(\varphi - \psi) \cos \frac{1}{2}(\varphi + \psi)}{2 \cos \frac{1}{2}(\varphi - \psi) \sin \frac{1}{2}(\varphi + \psi)} = \frac{1 - \tan w}{1 + \tan w} \quad \text{or}$$

(5) $\tan \frac{1}{2}(\varphi - \psi) \cot \frac{1}{2}(\varphi + \psi) = \cot (w + 45^\circ)$

(6) Letting $\tan \frac{1}{2}(\varphi - \psi) = \tan v$

(7) $\tan \frac{1}{2}(\varphi - \psi) = \tan v = \tan \frac{1}{2}(\varphi + \psi) \cot (w + 45^\circ)$

Since $\frac{1}{2}(\varphi + \psi) = w$ by formula 1
and $\frac{1}{2}(\varphi - \psi) = v$ by formula 6

From 7 we get

(8) $\tan v = \tan u \cot (w + 45^\circ)$

(9) $\varphi = u + v$

(10) $\psi = u - v$

This provides all the necessary elements for complete solution of the triangles involved.

A numerical example to illustrate the necessary computations follows (See Figure 10):

Given:	Point	X Coordinate	Y Coordinate
	A	496291.778	670 7421.871
	C	498821.693	670 7113.595
	B	498670.933	670 3512.131

$\alpha = 112^\circ\ 21'\ 13''.11$
$\beta = 100^\circ\ 06'\ 53''.55$
$\gamma = 94^\circ\ 33'\ 01''.40$

Find: X and Y coordinates of Point P

$u = 180^\circ - \frac{1}{2}(\alpha + \beta + \gamma)$

$u = 26^\circ\ 29'\ 56''.0$

$a = \sqrt{(2529.915)^2 + (308.276)^2} = 2548.628$

$b = \sqrt{(150.760)^2 + (3601.464)^2} = 3604.618$

$\tan w = \frac{a}{b}\frac{\sin \beta}{\sin \alpha} = 0.03267252$

$w = 1^\circ\ 52'\ 16''.8$

[illegible]

$\tan v = \tan u \cot (w + 45^\circ) = 0.46700985$
$v = 25^\circ\ 01'\ 58''.9$

$\varphi = u + v = 51^\circ\ 31'\ 54''.9$

$\psi = u - v = 1^\circ\ 27'\ 57''.1$

$$s = \frac{b \sin \psi'}{\sin \varphi'} = \frac{a \sin \psi}{\sin \varphi}$$

$$s = \frac{3604.612 \times 0.02558086}{0.17562232}$$

$s = 525.043$

$\psi' = PCB = \ = 180^\circ - (\varphi + \psi) = 78^\circ\ 25'\ 09''.3$

If t_1 = azimuth CB, then

$$\tan t_1 = \frac{X_B - X_C}{Y_B - Y_C} = \frac{-150.760}{-3601.464} = 0.0418608$$

$t_1 = 182^\circ\ 23'\ 49''.4$

If t_2 = azimuth CP

$t_2 = t_1 + \psi' = 260^\circ\ 48'\ 58''.7$

$X_p - X_c = 525.043 \sin 260^\circ\ 48'\ 58''.7$
$= -518.313$

$Y_p - Y_c = 525.043 \cos 260^\circ\ 48'\ 58''.7$
$= -83.797$

Therefore the coordinates of Point P are:
$X_p = 498821.633 - 518.313 = 498\ 303.380$
$Y_p = 67\ 7113.595 - 83.797 = 670\ 7029.798$

R. P. Bailey's solution for the three-point problem (Figure 11b) is

$$s^2 = \frac{s_1^{\ 2}\ s_2^{\ 2} \sin^2 (\alpha + \beta + \phi)}{s_1^{\ 2} \sin^2 \beta + s_2^{\ 2} \sin^2 \alpha + 2\ s_1\ s^2 \sin \alpha \sin B \cos (\alpha + B + \phi)}$$

and $E^2\ (u^2 \quad v^2)^2 = \{\ [(y_1{}' - X_1{}^1 \cot \alpha)\ F_1 - y_1{}^1\ (u^2 \quad v^2)]$

CONT

$$\csc^2\alpha \, d\alpha - (y_1^1 - x_1^1 \cot\alpha) F_2 \csc^2\beta \, dB)^2 +$$

$$s^2 \left[F_1 \csc^2\alpha \quad -F_2 \csc^2\beta \, d\theta \right)^2$$

where

E = approximate position error

$u = X_1 - X_2 + y_1 \cot\alpha + y_2 \cot B$

$v = y_2 - y_1 + x_1 \cot\alpha + x_2 \cot\beta$

$x_1^1 = x_1 \cos\theta + y_1 \sin\theta$

$y_1^1 = - x_1 \sin\theta + y_1 \cos\theta$

$F_1 = x_1 u - y_1 v$

$F_2 = y_2 v - x_2 u$

$d\alpha$ = error in observed angle α

$d\beta$ = error is observed angle β

$d\varphi$ = error is observed angle φ

5

ELECTRONIC POSITIONING SYSTEMS

The rise of electronic navigation is as rapid and spectacular as the progress of celestial navigation was slow. The beginning of the modern electronic industry stems from the publication in 1864 of James Clerk Maxwell's "Theory of Wave Travel". In 1883 Heinrich Hertz began his studies of Clerk Maxwell's electromagnetic theory which a few years later resulted in two discoveries which are the basis of radar and electronic positioning systems:

1) That radio waves are reflected by obstructions, and

2) That reflected radio waves obey laws of reflection, refraction, and propagation very similar to those followed by light rays.

In 1924 Breit and Tuve of the Carnegie Foundation measured the height of the Kennelly-Heaviside layer of the ionosphere by use of short length radio waves. In 1936 the U. S. Army developed the first pulse type radar system and one year later the first shipborne radar trials were held. In 1942 the development of the magnetron permitted the use of very high frequency waves known as micro-waves. The need during World War II for methods of long range navigation resulted in the development of the basically similar systems of Loran by the U. S. for ship navigation out to 500 miles and Gee by the British for aircraft bombing navigation to 300 miles. Since then many new improvements have been devised

and developed until we find now that each day brings some very significant advance in our knowledge of electronic positioning.

The principle of measuring distances by electronic methods consists simply in timing radio waves. The resultant accuracy, because it is a function of timing, is practically independent of distance. The propagation of radio waves obeys the formula:

$$\text{Wave length} = \frac{\text{Velocity of propagation}}{\text{Frequency}}$$

From this it can be seen that there are several obvious relationships which should be understood before proceeding with a detailed discussion of the various systems:

1. Low frequency waves have long wave length (long waves).
2. High frequency waves have short wave length (short waves).
3. Any variation in the velocity of propagation will result in a change of wave length.
4. Any change in the frequency will result in a change of wave length.

In discussing the characteristics of radio waves it is customary to use interchangeably the terms low frequency and long waves, and also the terms high frequency and short waves. Although this is quite proper, as can be seen from the first two relationships shown above, it sometimes tends to confuse those who do not employ the terms as a matter of course in their daily work. In this discussion when confusion may exist both of the synonomous terms will

be used, one being given in parenthesis. For example, low frequency (long waves).

FREQUENCY

The choice of frequency used in any system is a very important consideration because it is dictated by the requirements for size, accuracy, range, limitations of the equipment, frequency bands available, and assorted technical problems. The permanent assignment of a frequency for surveying purposes is just about impossible on an international basis because various countries have differently assigned frequency bands. Some of the systems already employ frequencies so close to the local broadcast or television bands that interference arises. Figure 13 gives a general idea of the bands assigned for different purposes. The characteristics of the extremes of frequencies are:

High frequency (short waves) are characterized by smaller size antennas; increased accuracy; line of sight range; less interference from sky wave effects since short waves are not reflected by but pass through the ionosphere; and exceedingly short waves will be reflected by any small object such as raindrops, reducing their usefulness during inclement weather.

Low frequency (long waves) are characterized by very large and very expensive antenna systems; less accuracy; ground wave transmission capable of circling the earth, which means possible positioning over the entire surface of the earth; sky wave interference; and increased noise level since atmospheric electrical

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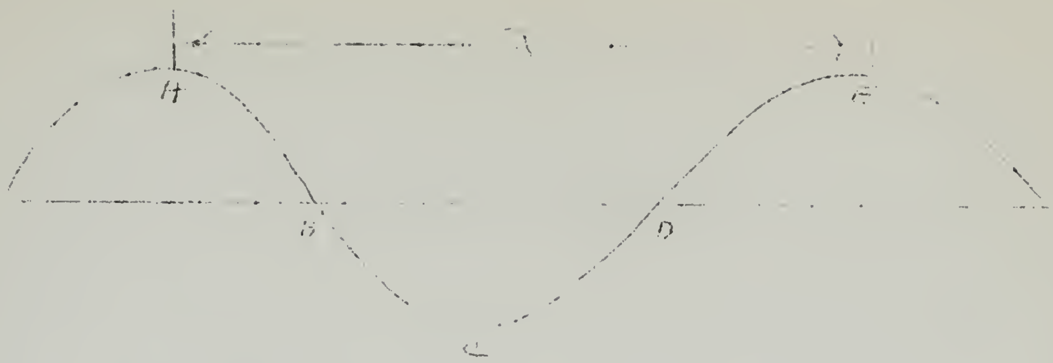
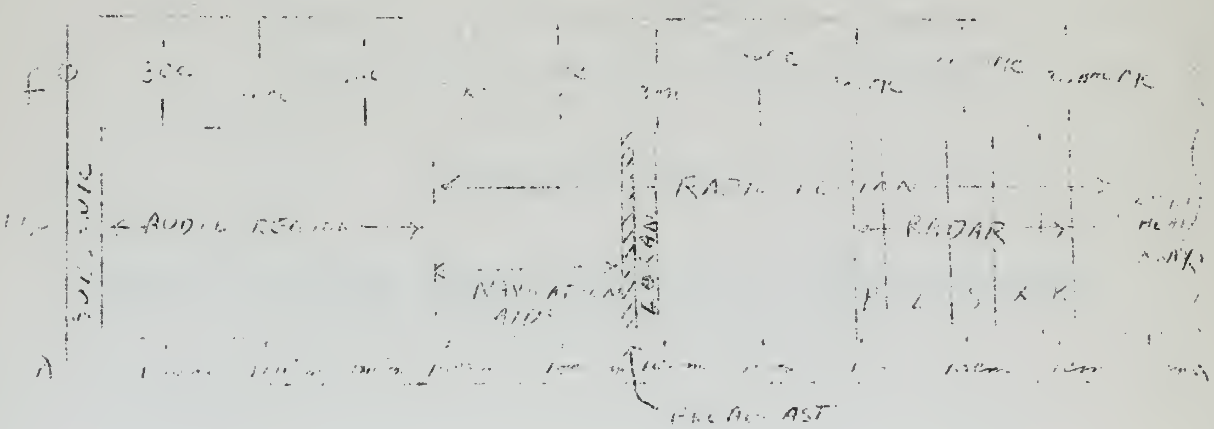


λ Is the wave length.
Cycle is the path followed (ABCDE) without repeating any angle.
Frequency is the number of cycles per given unit of time.

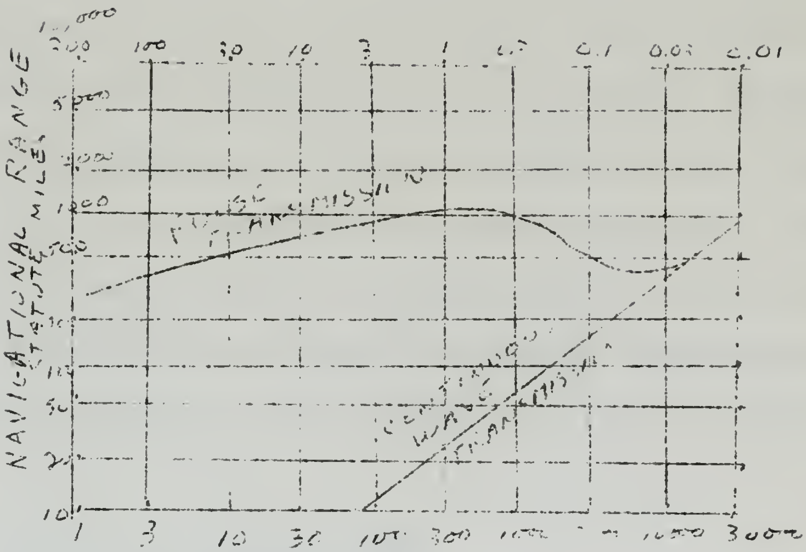
FIGURE 12: Relationships of wave length, cycle, frequency.



f = Frequency in cycles (c), kilocycles (kc), megacycles (mc).
λ = Wave length in meters (m) and centermeters (cm).

FIGURE 13: Frequency bands of basic spectrum.

Frequency in Megacycles



Wavelength in Meters

FIGURE 14: Maximum range of operation of a ground wave system without serious sky wave interference

discharges generate fantastic field strengths in the low frequency regions.

METHODS OF TRANSMISSION

In transmitting radio waves either pulse (intermittent wave) or continuous wave forms may be employed. Pulse power is by far the more economical of the two types since it requires much less mean power. In addition the ground wave may be resolved without interference from reflections from the ionosphere called sky waves. Pulsed systems can generally be worked in areas of high atmospheric noise where continuous wave systems would break down. However the accuracy attainable with pulse systems is much less than that obtained from continuous wave systems, especially those employing the principle of measuring phase angles to be discussed shortly.

METHODS OF TIME MEASUREMENT

There are various methods used in timing the radio waves, each with its own peculiar technical advantages and disadvantages:

Measurement of pulses is done by measuring the distance between two pulses. This can be done to about one-fifth of the pulse length being theoretically accurate to a least reading of 5 wave lengths. Assuming that the average error is $2\sqrt{2}$ times the least reading, a high frequency system of one meter wave length (such as Shoran) will give a least reading of 5 meters and an accuracy of about 14 meters (50 feet).

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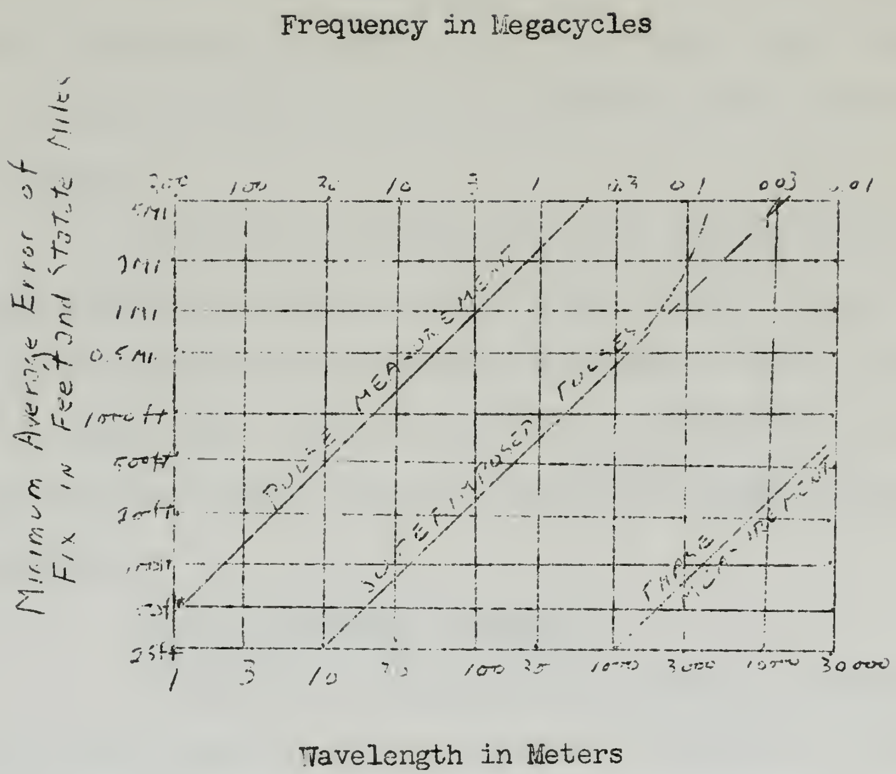


FIGURE 15: The minimum errors of fix attainable under the best geometric conditions with no transmission errors by three standard measuring techniques

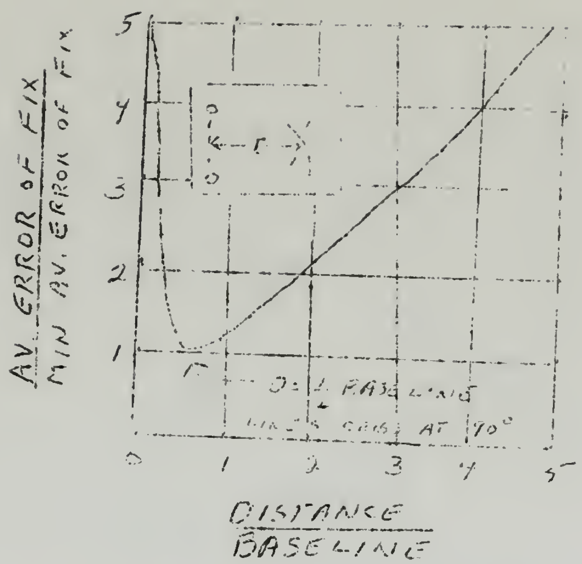
Measurement by pulse matching is the technique used, to date, only in the case of Loran and in a modified sense by EPI. It consists of making the amplitudes of two pulses equal in the receiver, and then visually superimposing and comparing them. This can be done with an accuracy of 1% of the pulse length which results in a reading 20 times more accuract than the direct measurement of pulses.

Measurement of phase consists in measuring the phase angle difference between two radio waves. It is the most accurate of all techniques with a theoretical accuracy of 1% of a cycle and 1° of phase angle. Figure 15 illustrates the accuracy attainable from all three methods depending on the wave length (frequency) employed.

CLASSES OF LINES OF POSITION

If position is determined by measuring the round trip travel time from the ship's transmitter - receiver to a fixed shore station a family of concentric circular lines of position is set up, with the fixed shore station as center. If position is determined by measuring the time or phase difference from two synchronized transmitting shore stations a family of hyperbolic lines of position is established with the transmitting shore stations as foci. In either system the intersection of two or more lines of position determines the position of the ship. This is known as fixing the position, or simply as a fix.

Circular systems, also called ranging or distance measure-



Example: At 3 times the length of the baseline the average error of fix is 3.1 times the minimum average error of fix.

FIGURE 16: The relative errors of fix of circular systems as a function of distance

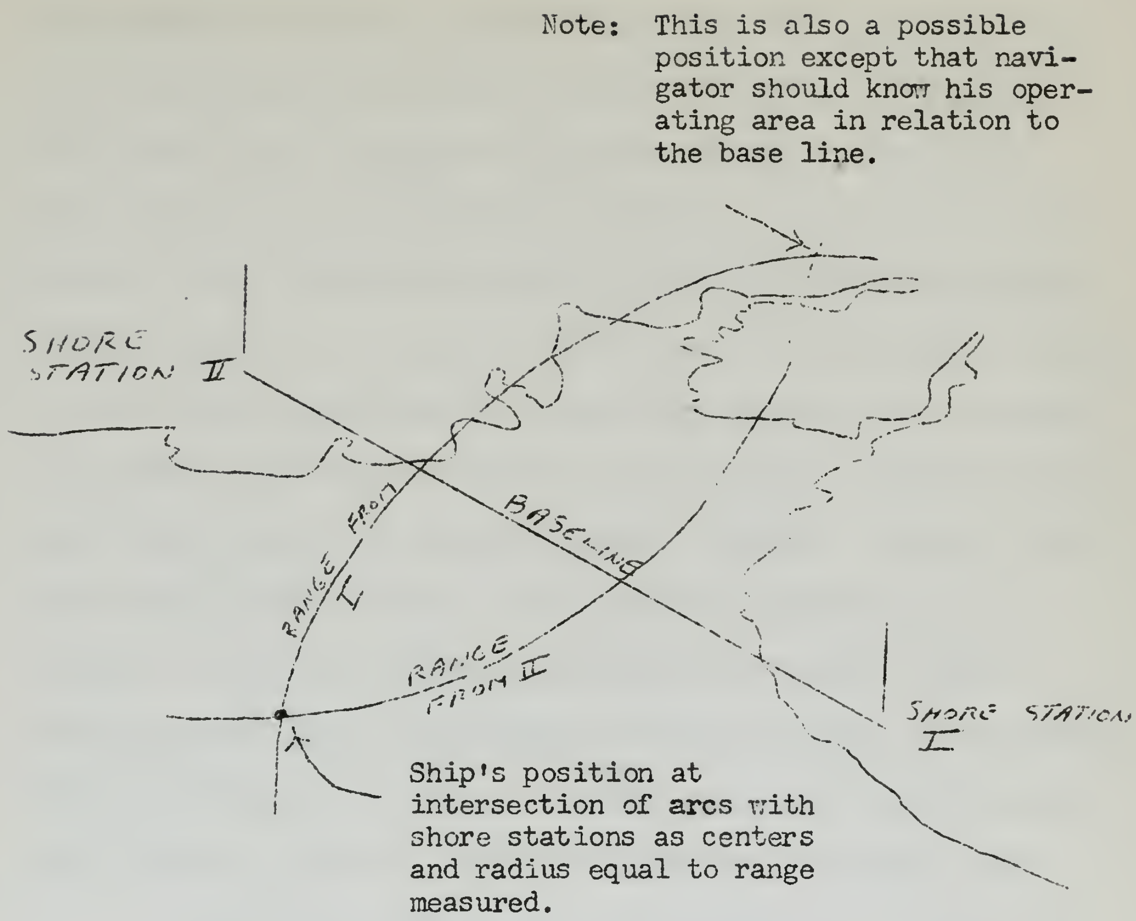


FIGURE 17: Principle of circular systems

ment systems, requires transmission by the user which seriously limits the number of users. If more than one unit is to operate in a given area with the same shore equipment and frequencies, an elaborate time-sharing schedule must be set up and adhered to. Under this type of operation a fix can only be obtained at the time permitted by the time-sharing schedule. Any attempts to get a fix at other times will result in interference with the signals of the other users. The other disadvantages of the circular system are that relatively elaborate transmitting and receiving equipment are required aboard the using ship; and only line of sight distances are obtained due to the high frequencies employed. However, there are several great advantages which sometimes outweigh the disadvantages: Circular systems are unaffected by clouds, darkness, or decreased visibility; no precomputed charts or tables are necessary; better geometrical accuracy is obtained; and there is no disruption due to sky wave effect. Figure 17 illustrates a circular system.

Hyperbolic systems employ time difference or phase difference to obtain a line of position. By definition, the locus of points, the difference of whose distance from two fixed points is constant, is a hyperbole. In electronic hyperbolic systems the measurement of a given time or phase difference fixes the observer's position on a hyperbole for which the two transmitting stations are the foci. Usually three transmitting stations are set up ashore with the center station acting as control for the two end stations. Such a set-up is called a triplet with the center

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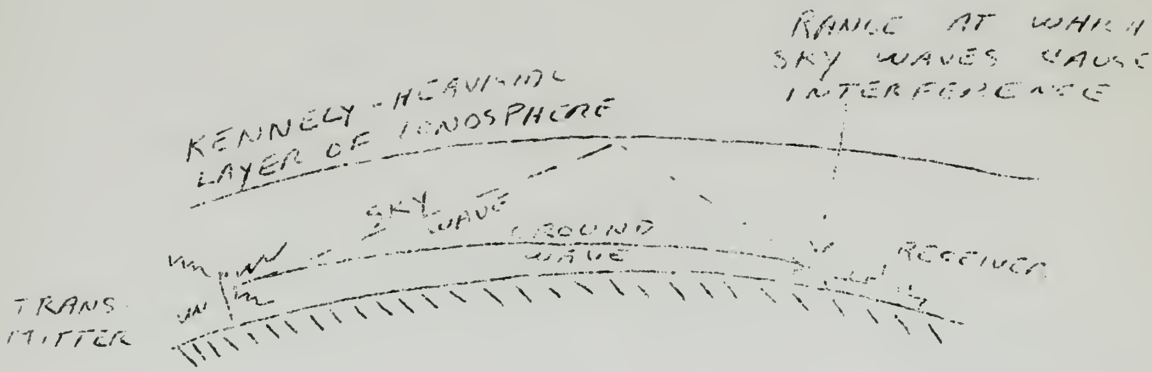
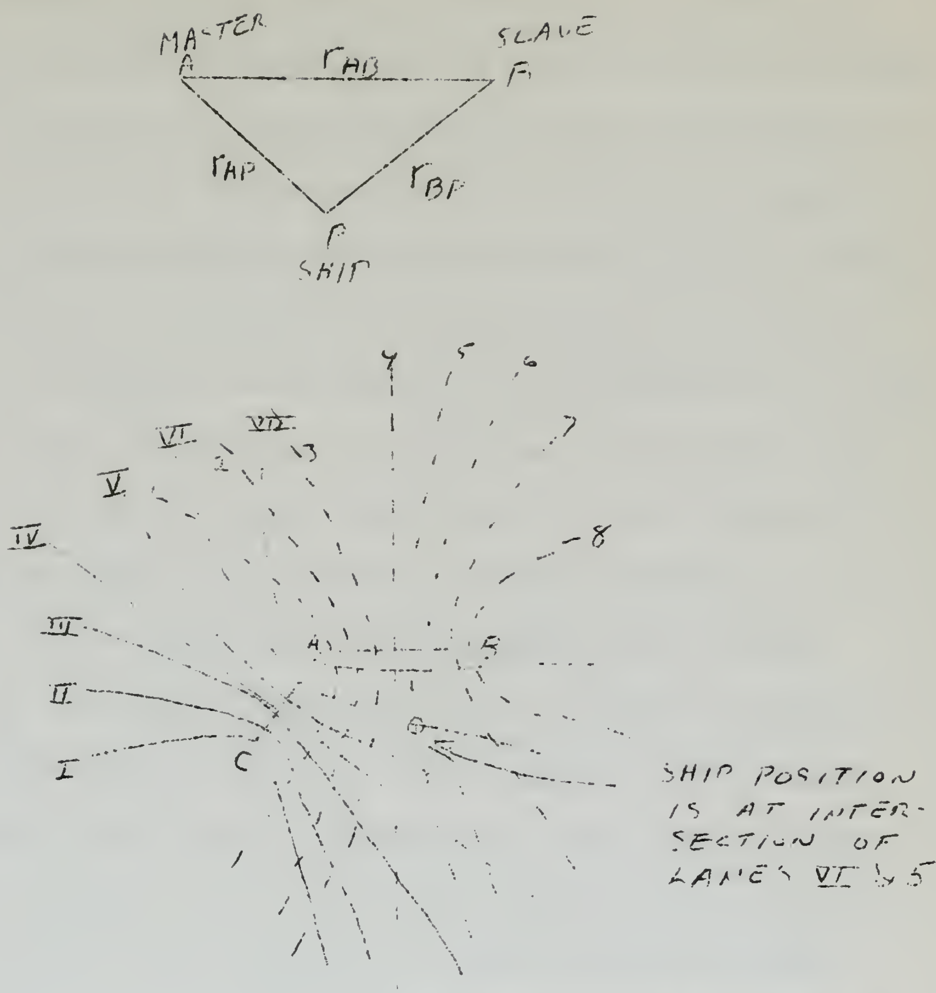


FIGURE 18: Ground and sky waves



Each hyperbole represents a line of constant time or phase difference.

FIGURE 19: Principle of hyperbolic systems

station called a master and the two end stations referred to as slaves. This results in a grid consisting of two families of hyperbolic curves, the intersection of any two curves determining the ship's position. A hyperbolic system has the advantages of longer ranges due to the low frequencies (long waves) employed; extremely simple shipboard equipment is required; and since there is no shipboard transmission, an unlimited number of observers may use the equipment at the same time. The disadvantages of the system are that interference or disruption of power causes loss of lane count which requires resetting of the dial readings at a point whose correct lane readings are known; interference from sky wave effects limits its use to daylight hours; and, most important, precomputed hyperbolic curves or tables are necessary before positioning can be done. This latter is a very serious disadvantage in the case of hydrographic surveys because, since the exact position of the three shore stations must be known before the curves can be computed, at least 50 miles of coastal triangulation must be completed before operations can commence. This results in long periods of inactivity for the sounding craft while the triangulation parties are obtaining the field data. Figure 19 illustrates a hyperbolic system.

Electronic systems can be classified in any one of the ways discussed above:

By the frequencies used.

By the type of transmission used (pulse or continuous wave).

By the method of time measurement (pulse measurement, superimposing pulses, or phase measurement).

By the type of lines of position obtained (circular or hyperbolic).

For the purposes of this report all systems will be classified into either the circular or hyperbolic categories with appropriate subdivisions as necessary.

The over all accuracy of an electronic positioning system depends on the instrumental, propagational, mapping, and observational errors involved. These errors can be separated into three main classes, internal, external, and geometrical.

Internal errors, composed of observational and instrumental errors, occur within the electronic equipment. Observational errors are closely connected with personal errors which were previously discussed in Chapter 2.

Instrumental errors are similar in principle to the instrumental errors of a sextant or other precise instrument and those errors that are common to all instruments will not be touched upon. However, there are special instrumental errors involved solely with electronic systems and which are of great importance in understanding electronic errors. A full discussion of all of the instrumental errors that affect the accuracy of an electronic system is beyond the scope of this report but a typical source of error occuring in hyperbolic phase matching systems is that arising from errors in transmission and receiving. In transmitting a radio

wave there may be errors of phase and/or frequency which are caused by variations within the set itself or are caused by lack of proper calibration. Any shift of phase in a hyperbolic system will result in a swing of the entire hyperbolic grid similar to the swing that arises in a triangulation scheme unless it is rigidly controlled in azimuth. Each manufacturer has a different means of solving this phase locking problem which will be discussed in the descriptions of each particular set.

The effects of changes in frequency on a hyperbolic system can be obtained from the formula

$$N = \frac{f}{v} \left(\Gamma AB + \Gamma AP - \Gamma BP \right)$$

where

N = number of hyperbolic lane

f = frequency

V = velocity of propagation

ΓAB = distance between master and slave

ΓAP = distance between master and ship

ΓBP = distance between slave and ship

let $Q = (\Gamma AB + \Gamma AP - \Gamma BP)$

$$N = f \frac{Q}{v}$$

Differentiating the above we get

$$dN = \frac{Q}{v} df$$

which by substitution gives

$$dN = \frac{df}{f} N$$

which shows how large a change in lane reading (dN) will result from a change in frequency (df).

For pulse type systems a typical but very serious source of error is in the timing circuits. In pulse type systems this consists of the precise determination of starting time of the initial signal. To accomplish this each manufacturer has some sort of zeroing technique whose purpose is to reduce all timing measurements to the same basic starting time. Unfortunately the zero correction is not constant but varies with the "rise time" of the signals. This is the time necessary for the signal to build up from zero time to its minimum useable level.

EXTERNAL ERRORS, composed of propagational errors, sky wave errors, and mapping errors, occur outside the equipment.

Propagational errors are caused by variations and/or uncertainties in the velocity of propagation of radio waves. The velocity of radio waves in a vacuum is constant (the latest and best value is 186282.42 mi./sec) but as waves pass through air their velocity is retarded by an amount dependent on the dielectric constant of the air which is a function of temperature, absolute pressure, and relative humidity of the air. Long waves in addition are affected by the conductivity of the terrain over which the wave travels. There is a gradual decrease with height in the dielectric constant which is responsible for a slight bending of the electromagnetic wave. We are now able to measure temperature, total pressure, and water vapor pressure of the air with

sufficient accuracy to reduce observations under the best conditions (to the vacuum value) with a resultant standard error of one part in 150,000, but this is for aircraft Shoran waves propagated above the surface of the earth. However, under less favorable conditions the standard error of a long wave may be between one and ten parts in 100,000. Over salt water the error may be one in 100,000 but over fresh water or over a mixture of different types of land the error will be considerably higher. Long waves for long distances follow the curvature of the earth and are greatly influenced by the atmospheric conditions and conductivity. The conductivity is very difficult to determine and must be solved before long waves (low frequencies) can be used for very precise work involving any overland transmission. For that reason it is imperative that stations for hyperbolic systems be so set up as to have the least possible amount of land intervening between the stations. The relationship of velocity in a vacuum and actual velocity through the air is expressed by the formula

$$V = \frac{Vo}{n}$$

where V is the actual velocity through the air Vo is the velocity in a vacuum, and n is the refractive index of that portion of the atmosphere through which the wave passes.

Sky wave errors are caused by the interference of radio waves reflected from the upper atmosphere. About 25-30 miles above the surface of the earth during the day, and 60 miles at night, is a conducting layer of ionized gas which prevents

electromagnetic waves from spreading out into space. The ionization is caused by the sun's ultra-violet radiation during the day and by the ozone layer at night. Short waves (high frequencies) are permitted to pass through this ionized barrier but long waves are reflected by this layer, known as the Kennelly-Heaviside layer, and cause considerable interference with the ground wave propagated along the surface of the earth. These reflected waves, called sky waves, are weak or completely absent during the day due to absorption in the ionosphere but are at a maximum around sunset and sunrise, maintaining a high strength level all through the night. The interference of sky waves at night stems from the fact that the strength of ground waves decreases as the wave gets further from the transmitter until eventually the ground wave blends with the static. Since there is more static at night there will be a consequent decrease in the range of ground waves at night. The decrease in ground wave strength is much greater than the decrease in sky wave field intensity and falls to such a point that the sky wave is equal to or stronger than the ground wave. At this point the sky wave begins to control the character of the signal, giving phase readings which are in no relation to the actual position. Pulse systems in the high and medium frequency range have a great advantage because by using short pulses ground waves may be resolved without sky wave interference. Below 1,000 kc., however, it is difficult to radiate a pulse sufficiently short to be received completely free of overlapping sky waves.

Although sky waves can be used to advantage in routine navigation with Loran, they are a hindrance in precise work. Sky wave effect takes over at a distance which increases as the frequency is lowered and as the ground conductivity is increased. The Decca system operating in the 100 kc band is affected by sky waves at 30 miles and is too inaccurate for precise work at 240 miles. The Lorac system operating in the two megacycles region broke down when the end stations were separated 80 miles. Figure 18 is a graphical representation of sky and ground waves.

Mapping errors are errors concerned with plotting the ship's position on the chart. It will be recalled that distortion of the chart paper can vary 1% due to changes in atmospheric conditions within the chart house. Because of the thickness of even the finest pencil point and errors of plotting instruments the plotting accuracy can not exceed 1.0 to 4.0 yards on a scale of 1:10,000 and is proportionately greater for smaller scales. In addition there are errors arising from drafting the hyperbolic curves on the chart. To avoid needless repetition the reader is referred to the appropriate paragraphs of Chapter 3.

GEOMETRICAL EFFECTS decrease the accuracy of electronic positioning systems, the loss of accuracy depending on the distance and bearing of the ship from the fixed shore stations. Determination of this effect requires a knowledge of the geometry of the lines of position for the system involved. Of the two basic systems, circular and hyperbolic, the circular systems have by far the better

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

geometrical characteristics. In circular systems the distance between stations should be equal to the maximum coverage desired in order to get the best accuracy. Figure 16 indicates the effects of geometry on a circular system.

Hyperbolic systems have considerably larger geometric effects. It can be seen from Figure 19 that the center line of the hyperbolic grid formed by two shore stations (a slave and a master) is a straight line. Hyperbolas in the vicinity of the center line curve away very slightly, but hyperbolas in the vicinity of the foci (shore stations) curve very sharply. This diverging characteristic of adjacent hyperbolas gives rise to a varying position error in hyperbolic systems. If an error is made in determination of the time or phase difference, an observer would obtain his position as lying on an adjacent hyperbola. The resultant error in position will vary depending on the amount of divergence of the adjacent hyperbolas in the region of the observer's true position. Positional errors resulting from incorrect time or phase differences increases with the distance from the baseline and with approach to the baseline extension. The errors increase nearly as the square of the distance because the hyperbolas diverge while the angle of intersection of the two families of hyperbolas decrease. Figure 20 shows the effects of geometrical errors on hyperbolic systems. It should be noted that no point in the area covered by a hyperbolic triplet system has an error less than 1.3 times the minimum average error. For

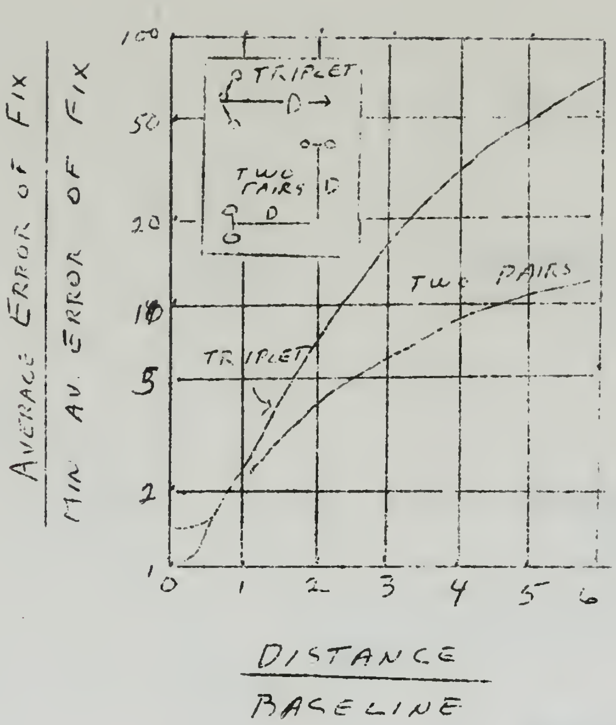


FIGURE 20: The relative errors of fix of hyperbolic systems as a function of distance (ground waves only)

[illegible]

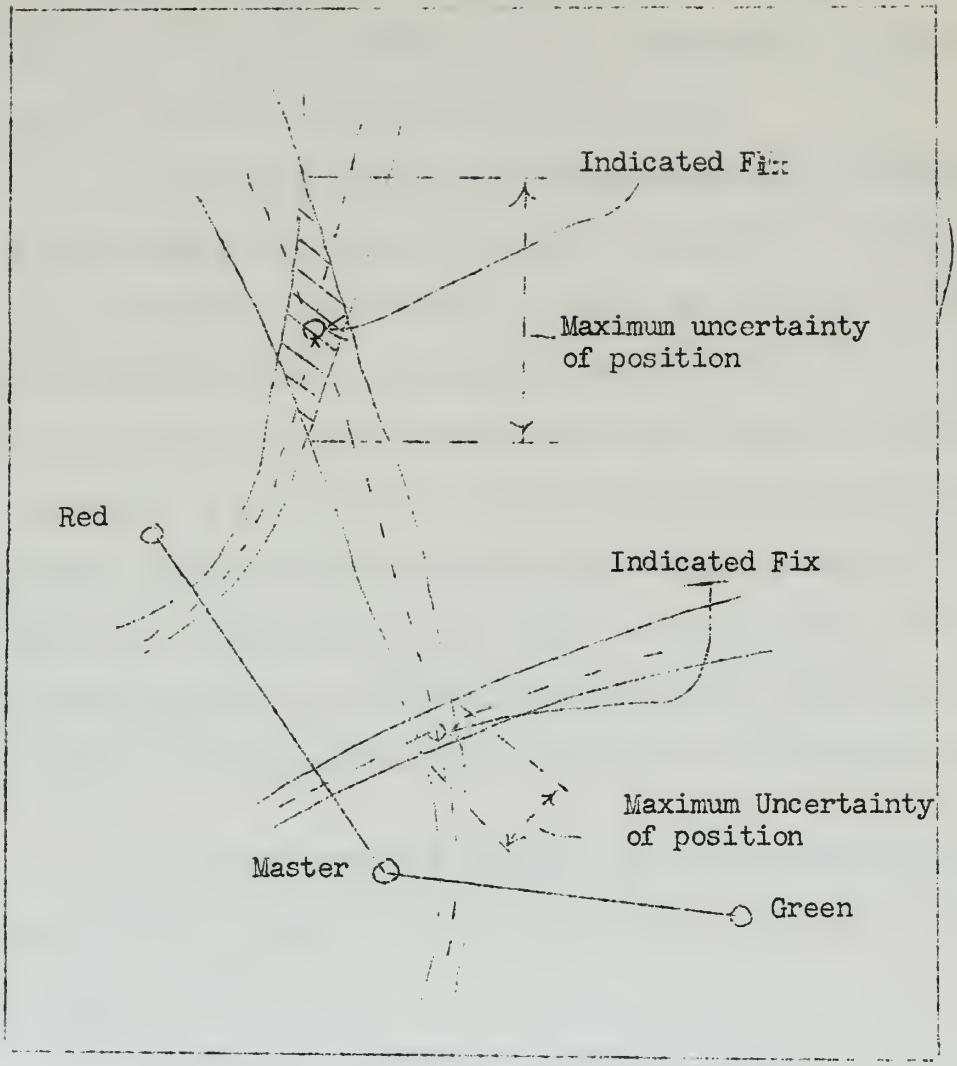


FIGURE 21: Relation of position and possible accuracy with hyperbolic system

optimum results in a hyperbolic system the base lengths should be NOT less and preferably greater than the coverage desired. However, it may be necessary to sacrifice baseline length to avoid sky wave effect. Figure 21 illustrates the uncertainty of position in various sectors a typical hyperbolic system.

It has been pointed out that the angle of intersection has great effect on the resultant error of position. An intersection of 90° will give the best results, but the angle of intersection should seldom be permitted to decrease below 30°. An inspection of Figure 4 will show how the error ellipse is affected by the angle of intersection. Another obvious error that should be kept in mind is that the accuracy of the ship's position depends on how accurately the shore stations were located. This is particularly true of hyperbolic systems where small errors in station position produce large errors in the hyperbolic grid.

All major surveying agencies now employ electronic systems to an increasing extent. The systems used by a few of the many surveying agencies are:

Shoran: U. S. Coast and Geodetic Survey, U. S. Air Force; and Canadian Department of Mines.

E. P. I.: U. S. Coast and Geodetic Survey.

Decca: British, Finnish, Swedish, and many other foreign governmental agencies.

Lorac: U. S. Navy Hydrographic Office, Bell Telephone Company.

Type E Raydist: Norfolk District U. S. Army Corps of Engineers; U. S. Air Force.

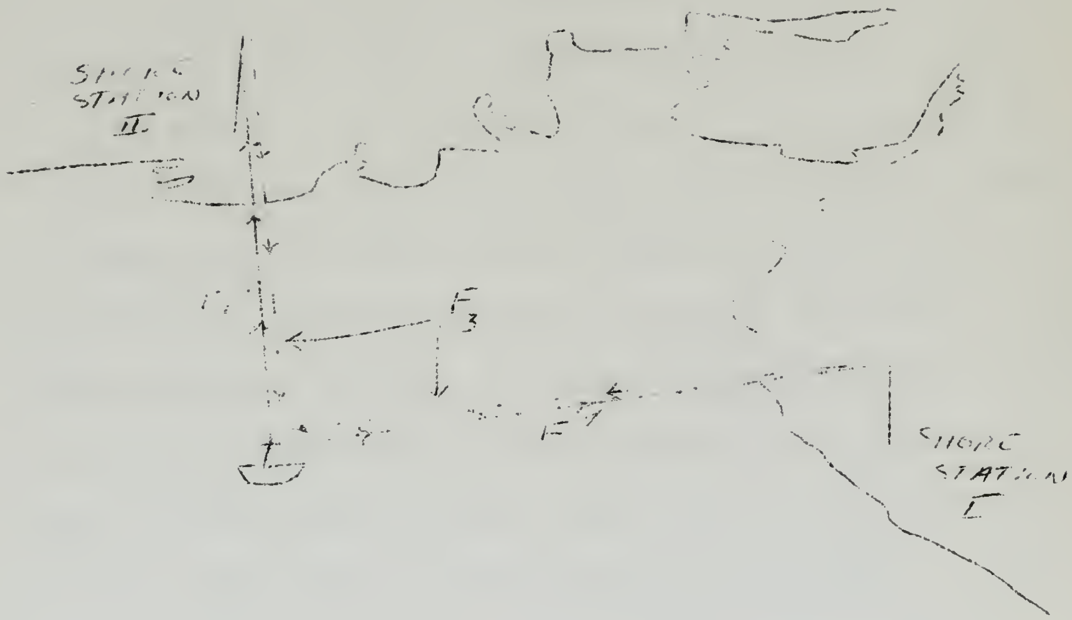
Loran: Merchant and naval vessels throughout the world.

Of the above only Decca and Shoran of the precise systems have been fully tested to determine the accuracy of the system. Loran has been tested but is not sufficiently accurate for precise work. So far as is known only one study has been made to compare the cost of carrying out a survey by electronic means as against visual means. This study is based on fragmentary data obtained over a very short period of time with untested experimental equipment. It did indicate, however, that if electronic break-downs can be held to 10% of the operating time, a considerable savings will result by using electronic methods instead of visual. Therefore a great deal more effort should be spent in fully testing the various electronic systems including a comprehensive time-cost study.

A beginning has been made by compiling Table III but the results and conclusions obtained should be followed with considerable caution because most of the data is either too sketchy to be conclusive or is based on the author's limited experience and personal prejudices. One should keep in mind the old adage to the effect that an observer who is attempting to prove a theory cannot be sufficiently objective to produce absolutely trustworthy data. The data will be slanted in the direction of proving the theory regard less of how hard the observer tries to maintain perfect objec-

tivity.

It is only necessary in summary to point out that electronic positioning systems offer tremendous advantages over visual methods in the ranges obtainable. There are also overwhelming indications that electronic systems are more reliable and more accurate. All things considered, electronic methods show great promise of producing more accurate and more dependable positioning than conventional methods for both offshore and inshore hydrographic surveying. The ability of electronic systems to give continuous and accurate positions several hundred miles from shore, independent of conditions of visibility, are factors of the greatest importance.



F_1 = low frequency from ship
F_2 = high frequency from ship
F_3 = 290 - 330 mc back to ship

FIGURE 22: Shoran System

6

SPECIFIC ELECTRONIC POSITIONING SYSTEMS

A. SHORAN

Shoran, standing for SHOrt RANge Navigation, is manufactured by the Radio Corporation of America (RCA) and is a high frequency (short wave) circular system based on measuring the time required for a radio pulse to travel from the aircraft or ship to the fixed shore beacons and back. It was originally developed in 1942 as a navigational aid to enable a bomber to strike its target when the target was not visible from the aircraft. As often happens, peaceful applications arose from the results of this wartime research. Shoran is now used for aerial electronic geodetic measurements, positioning of aerial photography for mapping purposes, and positioning of survey ships at sea. A new and considerably more precise model was developed and called HIRAN to differentiate it from the old model. Hiran was abbreviated from the term "high precision Shoran" and has been misinterpreted by many as being entirely new systems, but this is not correct. Shoran connotes a system of transponded high frequency electronic impulses and Hiran is only a higher precision model of the same basic Shoran system. Since Hiran is usually used for air applications, this discussion will confine itself to the basic Shoran system as used in hydrographic surveys.

A simplified presentation of the elements of the Shoran system is depicted in Figure 22. The basic equipment consists of

[illegible]

[illegible]

[illegible]

[illegible]

a transmitter and receiver aboard the ship and two stations located at known points ashore. One shore station is designated as the low frequency station to receive the 225-240 MC pulses, and the other as the high frequency station to receive the 240-255 MC pulses. Both shore stations retransmit a common frequency between 290-330 MC which are received at the ship from both shore stations. Even though identical frequencies are received from two widely separated shore stations there is no chance of confusing the pulses from each station because an entire cycle (as will be shown later) is 2/20 of a second. Since radio waves travel approximately 186,000 miles per second, in one tenth of a second they will travel 18,600 miles. Round trip distances are used, so confusion between pulses will only arise at ranges of 9,300 miles which is far in excess of the effective range of Shoran equipment. The distances from the ship to each shore station are measured in the following manner:

The ship transmits low frequency pulses for a period of one twentieth of a second. The transmitter is fired 1.79 microseconds before the zero reference mark appears on the cathode ray tube. This is to compensate for the time for the pulse to pass through the shore station. The pulses travel to the LOW frequency shore station where they are received, amplified, turned into high powered pulses, and after a total period of 1.79 microseconds retransmitted on a frequency between 290-330 MC.

The receiver on board the survey ship receives these pulses and sends them into the indicator position of the equipment. The indicator measures the transit time of the pulses and represents

[illegible]

that time in statue miles. The measurement can be made to a thousandth of a mile (5.28 ft) and the accuracy of the measurement is good to $\pm$ 50 feet. The high accuracy of Shoran is produced through the use of short pulses, with steep leading edges, the pulse duration being on the order of one-half microsecond. The pulse repetition rate is 931.09 C/Sec/100 miles with a transmitting frequency range from 225 to 330 MC.

During the next one-twentieth of a second the same procedure is used with the ship's transmitter of high frequency sending pulses to the high frequency shore station. The presentation in the cathode ray tube , being alternated every one-twentieth of a second, has the appearance of the two measurements being done simultaneously. This is due to the retention characteristics of both the fluorescent cathode ray tube and the human eye.

Arcs of circles with radii equal to the Shoran linear distances from ship to shore station are swung with the appropriate shore station as center. The intersection of the arcs is the position of the ship. There is a possible ambiguity of position because there will be two places of intersection, one on either side of the baseline connecting the two shore stations. This is of little concern to the navigator because the stations are widely separated and the navigator should know where his operating area is in relation to the baseline. However, should confusion exist the speediest way to resolve any ambiguity is to set course to cross the baseline. The plot of several fixes will quickly solve the problem.

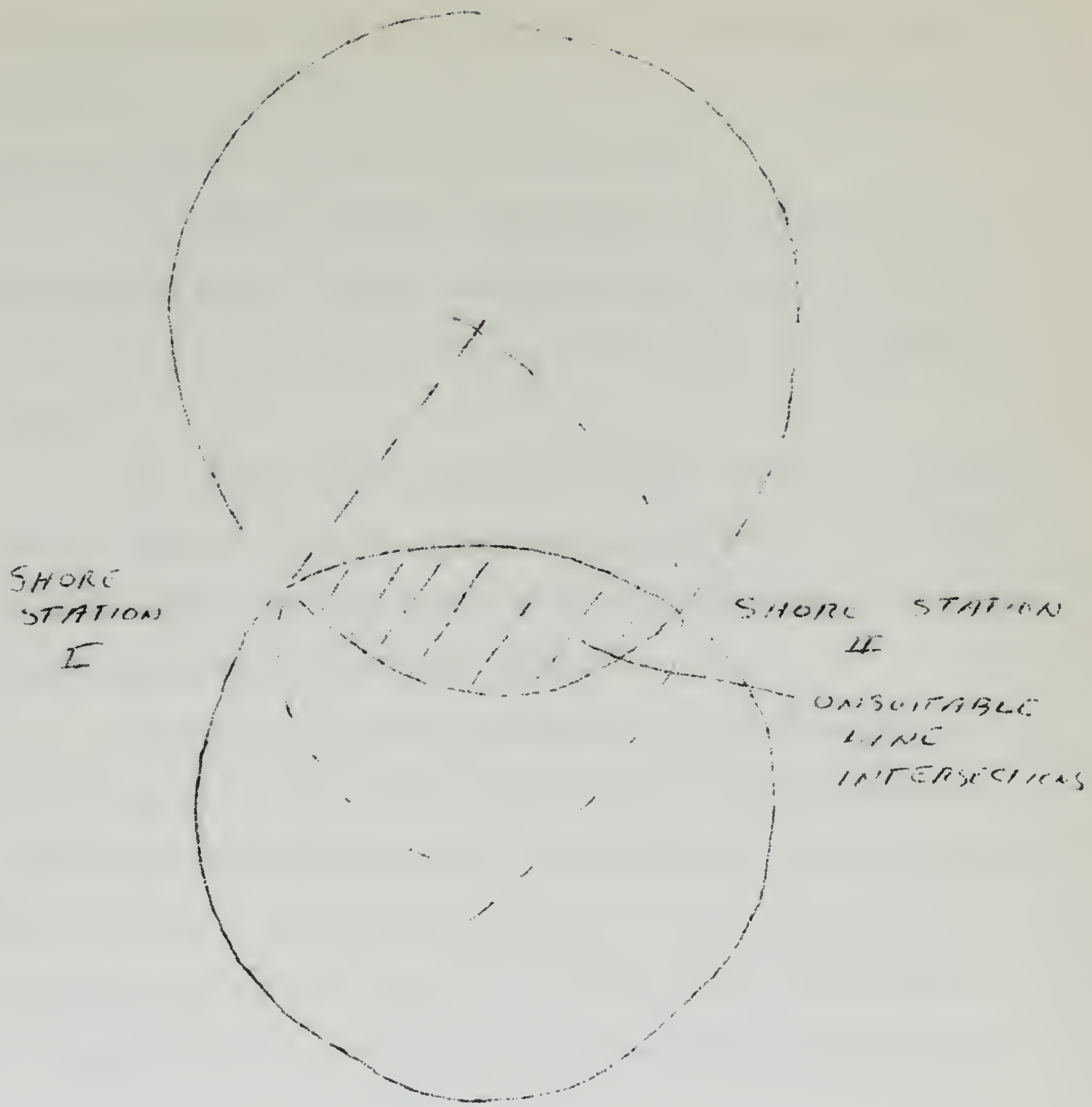


FIGURE 23: Circular System coverage for angle intersection of 30° to 150°

Although circular systems are not as seriously affected by geometric conditions as are hyperbolic systems, it is necessary to insure that the angle of intersection of the circular lines of position will be between the favorable limits of 30 and 150 degrees. This may be done as follows (see Figure 23):

1. Swing an arc of a circle from each station using a radius equal to the distance between the two stations. The two points where the arcs intersect are the 60 degree station angle points.

2. Swing circles from the 60 degree station angle points using a radius so that the circles will pass through each station. Any ship outside the area common to both circles will have an intersection angle between 30 and 150 degrees.

Shoran has been used successfully for hydrographic surveys by the U. S. Coast and Geodetic Survey and for geodetic and photogrammetric purposes by many other American and foreign government agencies. It is extremely accurate and reliable method of positioning for line of sight ranges, but has the disadvantage of a limited number of observers due to transmission by the user.

SPECIFIC ELECTRONIC POSITIONING SYSTEMS

B. E. P. I.

EPI, standing for Electronic Positioning Indicator, is manufactured by the U. S. Coast and Geodetic Survey. It is a low frequency (long wave) pulse type circular system combining the low frequency time measuring techniques of Loran with the circular distance measuring features of Shoran. In 1941 the U. S. Coast and Geodetic Survey began experiments with EPI to attemp to measure distances beyond the range of Shoran. It was first tested in 1947 and has been used in hydrographic surveys since 1950. This system uses a transmission frequency of 1,850 KC, a pulse duration of about 30 microseconds, and a pulse repetition rate of 41 2/3 pulses per second. The minimum practical range is 14 miles and dependable ranges under low static conditions can be had out to 500 miles. Bad static will reduce maximum ranges to 200 miles. Since static is worse at night, the standard procedure is to survey the outlying places during the day and inboard area at night.

EPI is a unique circular system in that it requires mobile ship equipment capable of transmitting LOW frequency pulses and measuring small increments of time which are converted to linear distances by special conversion tables; and two fixed shore stations capable of transmitting similar pulses accurately synchronized with those from the ship. Synchronization is obtained at the instant of fix by equalizing and superimposing the pulses at the shore

[illegible]

[illegible]

[illegible]

[illegible]

station. The ship receives the synchronized signals from the shore stations, measuring in microseconds the round trip travel time from ship to shore station. (The microseconds are converted to equivalent linear distances by means of conversion tables.) With the positions of the two shore stations accurately known and plotted on the chart, the ship's position can be determined graphically by swinging arcs similar to the method used in Shoran.

The reciprocal of a pulse repetition rate of 41 2/3 pulses per second is equivalent to a distance of 24,000 microseconds. The maximum scale reading without repetition is 9,999.9 microseconds but 6,000 microseconds is the practical maximum range. This corresponds to 550 miles or 480 nautical miles.

EPI is relatively untested but during simulated field tests of 267 observations and 21 observations under laboratory conditions a probable error of ± 0.2 microseconds (± 100ft) was determined. It has been used with excellent results by the U. S. Coast and Geodetic Survey in hydrographic surveys of the Gulf of Mexico and Bering Sea as well as for measuring geodetic distances in the Alaskan areas. Combined with Shoran for inshore hydrography, EPI will provide excellent accuracy and coverage for offshore hydrography out to 500 miles.

SPECIFIC ELECTRONIC POSITIONING SYSTEMS

C. LORAN

Loran, standing for LOng RAnge Navigation, is a pulse type hyperbolic system developed by the M.I.T. Radiation Laboratory during the World War II for the purpose of providing a long range electronic positioning system for ship convoys operating in the North Atlantic. At present there is complete coverage of the North Atlantic and the North Pacific Oceans with additional coverage contemplated. The stations are operated and manned for the most part by the U. S. Coast Guard, the British Royal Air Force, and the Royal Canadian Navy in conjunction with other agencies of the governments of the U. S., Canada, Iceland, Great Britian, and Denmark. The special Loran tables and hyperbolic charts of foreign waters required in order to use Loran are prepared and distributed by the U. S. Navy Hydrographic Office, Suitland, Maryland; hyperbolic charts for areas bordering the United States and its possessions are provided by the U. S. Coast and Geodetic Survey. The only equipment needed aboard ship to carry out Loran navigation is a Loran receiver and Loran charts or tables.

Loran, operating on four adjacent frequencies between 1,750 and 1,950 KC, provides hyperbolic patterns of lines of position out to distances of 1,400 miles at night by use of sky waves and to 800 miles during the day with ground waves. It has an accuracy of ±0.2% of the range to the station. This system utilizes accurate measure-

ments of time differences between the arrival of two radio pulse signals transmitted by a pair of shore stations separated from each other by a distance of 250-350 nautical miles.

Numerous station pairs (a master and a slave) can use repetitions of the same four frequencies by transmitting a slightly different number of pulses per second. Thus the frequency and pulse repetition rates are the distinguishing characteristics of the various station pairs. By three switch settings the Loran receiver can be set to any of the frequencies and can be synchronized with any of the pulse repetition rates.

The incoming signals are viewed on a cathode ray tube, after suitable electronic manipulation and magnification so that the pulses can be clearly seen. The two signals from one station pair are stopped on the cathode ray tube and isolated from signals from the other stations. The two pulses are then superimposed and the left sides of the pulses carefully matched. The time delay required to superimpose the signals can be determined by readings on three calibrated scales on the older models or by reading a dial similar to a speedometer on the newer sets.

An important principle of Loran is that the signals are not transmitted simultaneously from the master and slave stations. The slave signal is not transmitted until the corresponding master signal has been received at the slave station. Then an arbitrary but accurately controlled delay is added so that the master signal is always received first regardless of the position of the ship.

Synchronization is maintained with a tolerance of $\pm$ 2

microseconds. If for any reason this tolerance cannot be maintained, the transmitted signals are blinked on and off to warn navigators that the signals are not synchronized. Separate monitor stations also keep continuous watch on the transmission. Only severe static (such as magnetic storms and heavy precipitation static) or electronic failure can hamper or prevent use of Loran.

For those who may be interested in the simple mathematics behind the above discussion, Figure 24 should be examined. The number of microseconds it takes a radio wave to travel from the master to the slave station is denoted by δ

> T_B is the time required to travel from the slave station to the ship.
>
> T_A is the time required to travel from the master station to the ship.
>
> $T_B - T_A$ is the time difference, + if the ship is nearer station A and − if nearer station B.

Since the pulses have identical electronic characteristics it is impossible in practice to determine the sign of the time difference and this introduces an ambiguity. In hyperbolic systems a given time or phase difference will locate the ship on either of two hyperbolas symmetrical to the center hyperbola which is a straight line. In Loran this ambiguity is resolved by having the slave station transmit later than the master.

Let the interval of time from the emission of a pulse from the master to the emission of the next pulse from the slave be the absolute delay, D. Then the time difference at the ship TD is greater

or less than D by the amount station B is nearer or farther from the ship than Station A. Therefore,

$$TD = D + (T_B - T_A).$$

Depending on where the ship is in relation to the master or slave stations the quantity $(T_B - T_A)$ can vary from $+\vartheta$ to $-\vartheta$. Therefore, if D is greater than ϑ, TD will always be positive. This means that the pulse from the master, A, would always be received before the pulse from the slave, B.

To identify the received signals as coming from the master or slave, the slave maintains its transmitted signal in synchronization with that from the master station. For instance, if the time sequence as alternately emitted by a pair is ABABAB etc, then the interval between successive A's or successive B's is called the repetition interval, L. The transmission interval is AB and corresponds with the absolute delay, D, which must be less than L. The reception interval at the ship, AB, is the same as the time difference TD which varies with the location of the observer.

To prevent ambiguity in identifying the received signals as coming from the master or slave, D is made of such length that

$$L > T > D > \frac{L}{2}$$

This causes the interval between the reception of the master signal A and reception of the slave signal B to be everywhere greater than the interval between reception of the slave signal and reception of the next master signal. In other words, it makes certain:

1) The master is on the upper trace and the slave is on the lower trace and 2) the slave signal and the NEXT master signal cannot

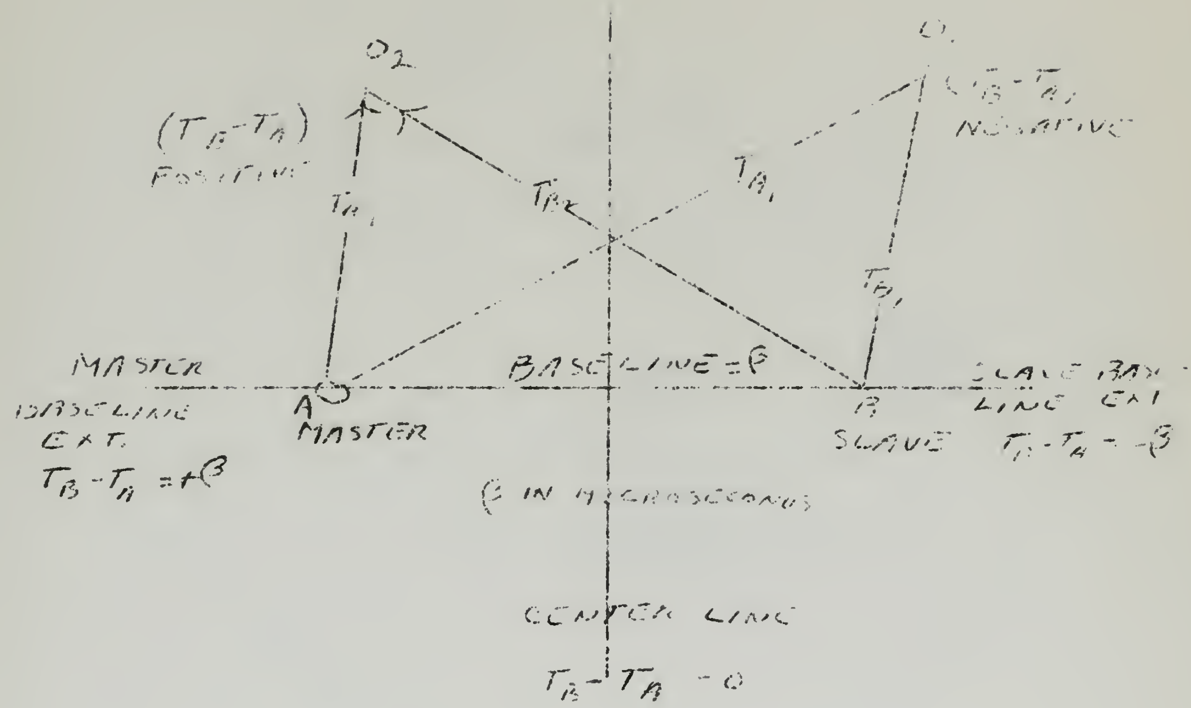
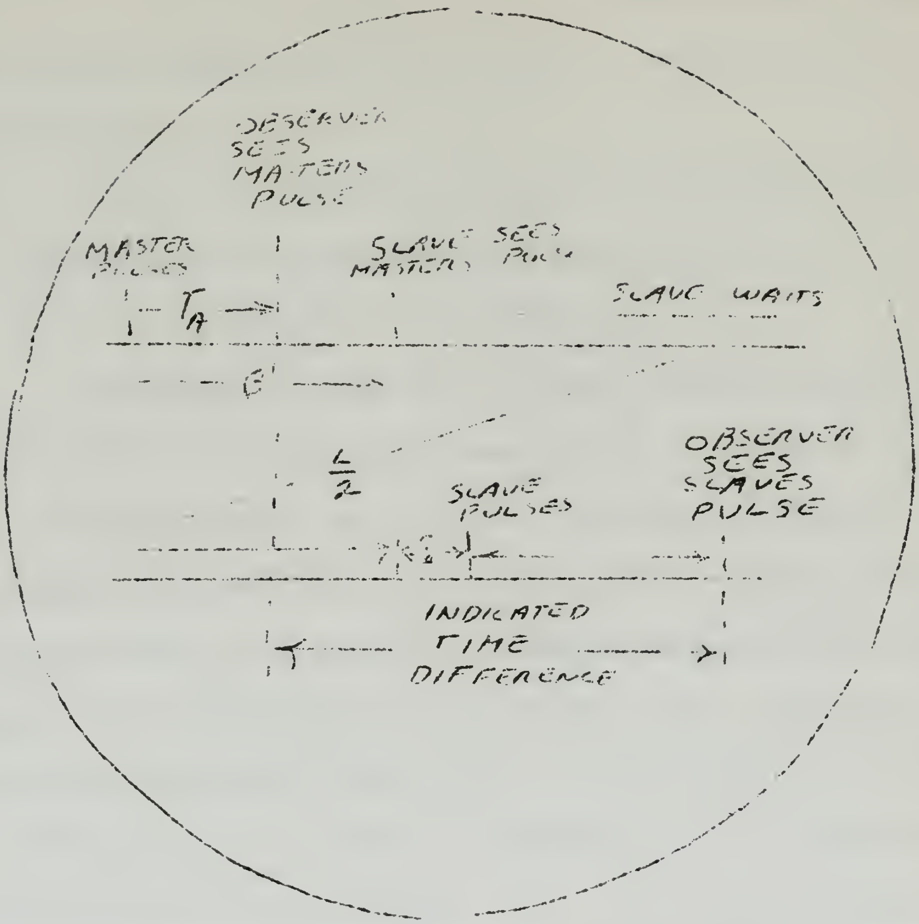


FIGURE 24: Derivation of Loran equation



$$ATD = \frac{L}{2} + \beta + \delta + (T_B - T_A)$$

$$ITD = \beta + \delta + (T_B - T_A) \quad \text{GENERAL}$$

$$ITD = 2\beta + \delta \quad \text{at master}$$

$$ITD = \delta \quad \text{at slave}$$

FIGURE 25: Loran Scope presentation

appear on the lower trace. See Figure 25. This can be written in the form of an equation so that the following conditions must be fulfilled:

$$D - \beta = \frac{L}{2} + \delta$$ where δ is some arbitrary time interval within the limits 0 to $\frac{L}{2} - 2$ and is known as the coding delay.

Therefore, the absolute delay, D, equals $\frac{L}{2} + \beta + \delta$ and the fundamental Oran equation may be written:

$$ID = \frac{L}{2} + \beta + \delta + (T_B - T_A)$$

The complete cycle of events can be followed from Figure 25 which shows the actual scope presentation. Each trace is equal to $\frac{L}{2}$. The split trace presentation cancels the L/2 value used in the formula for Actual Time Difference (ATD). The Indicated Time Difference (ITD) is the difference actually measured on the scope. Readngs at the master must be equal to $2\beta + \delta$ because T_A will be 0 and T_B will be equal to β. At the slave end T_B will be 0 and T_A equal to β to give a reading of δ. If the reading at the master station is not $2\beta + \delta$ or if the reading at the slave station is not δ then the system is not in proper synchronization and the signals will begin automatically to blink on and off. The slave station then makes the necessary adjustments to its timing equipment to restore the prop r readings. There is thus a constant check on its service accuracy since if the readings are correct at the master and slave stations, they will be correct throughout the service area. If the readings are not correct the pulses are blinked as a warning to navigators to not use the signals.

Due to the great distances involved in Loran navigation corrections must be applied in constructing the hyperbolic grid in order to allow for departure of the earth from spherical to oblate spheroidal shape. These corrections become necessary at ranges of about 100 miles. The procedure is to convert the plane hyperbolas to spherical hyperbolas which are then corrected for oblateness of the earth by suitable correction factors. All of the necessary computations are done by high speed electronic computers, another great advance made by the electronic industry.

Although Loran is not sufficiently accurate for precise coastal hydrographic purposes, it is an excellent method for routine navigation on the high seas. It is also extremely useful for hydrographic surveying in the open ocean where more precise control is not available. It is used to good advantage by ships controlled by the U. S. Navy Hydrographic Office for control of oceanographic surveys and for developing submarine topography by echo soundings. The inaccuracies of the system are due to the design characteristics of the electronic components. Improved design can increase its accuracy theoretically to $\pm$ 3.2 nautical miles (see Figure 14, 15, and 20), but would probably cause it to lose most of its value as a navigational method due to loss of simplicity of operation and ease of maintenance.

SPECIFIC ELECTRONIC POSITIONING SYSTEMS

D. DECCA

Decca is an electronic system manufactured by the Decca Navigator Company, Ltd., 1-3 Brixton Road, London S. W. 9, England. It is based on the principle of measuring phase differences between continuous wave radio signals received from a master and slave station to give a resulting hyperbolic line of position. Many of the principles involved in Decca apply to other continuous wave hyperbolic systems, such as Lorac and Raydist. These principles will be explained in this section but will not be repeated in the later sections dealing with Lorac and Raydist.

The Decca system was invented in 1942 and put into production after the end of World War II. It has been tested by various departments of the British, Danish, Dutch, Finnish, French, and Swedish governments, providing a detailed evaluation of the possibilities of using frequencies of the order of 100-340 KC for short and long range navigation. For routine navigation purposes the area of coverage is not as great as Loran, being 240 nautical miles at night and 600 nautical miles by day. However, it is extremely precise, has excellent repeatability characteristics, and is being used to great advantage for precise surveying.

Coverage for the Decca system is provided by three stations: a central master and two flanking slave stations. In a typical pair the phase of the continuous wave transmissions from the slave station is locked to that of the received master signal

and thereafter phasing is maintained automatically. In order for this to be satisfactory a relatively strong ground wave is required from the master, which limits the distance between master and slave stations to 70 miles. The method of locking the slave's signal to that of the master's is to have the master and slave always transmit out of phase with each other in such a way that the master's radiating phase is $360^\circ - \phi_A$ where ϕ_A is the phase arriving at the slave station. This triggers the slave so that it will radiate at a phase of $360^\circ - \phi_A$. When this wave from the slave arrives at the master station it will be in phase with the master, locking the system to give a rigid pattern of hyperbolas. To correct any phase variations that may arise a special monitor station is set up on an island or in a distortion free area away from any stray radio waves or other sources of electronic inference that may introduce errors. The operator at the monitor station can change the phase of the red or green slave stations so that the pattern will be constant. This insures complete phase locking.

In order to compare the phases of the received signals, each station transmits upon a separate frequency which is multiplied by the Decca receiver in such a manner that the incoming signals from each pair are automatically measured at the lowest common harmonic called the comparison frequencies. These frequencies employed are:

STATION	TRANSMITTED FREQUENCIES	MULTIPLICATION FACTOR	COMPARISON FREQUENCY
Master	85 KC	4	340 KC
Red Slave	113.5 KC	3	340 KC

STATION	TRANSMITTED FREQUENCIES	MULTIPLICATION FACTOR	COMPARISON FREQUENCY
Master	85 KC	3	255 KC
Green Slave	127.5 KC	2	255 KC

The receiver comparison frequencies are made higher than the transmitted frequencies by the factors, 4, 3, 3, and 2 in order to get more accurate measurements for reading phase differences, because the lane separation for higher frequencies is proportionately less which reduces the final errors in measurement. The manufacturer claims that a Decometer is capable of distinguishing phase differences of about 3 degrees which is about 1/100 of 360 degrees. This results in the system being able to distinguish a difference of 1/100 of the wave length of the comparison frequency. Along the base line this results in an accuracy of about ± 15 feet.

One complete cycle of 360 degrees is termed a lane, and is equivalent to a distance of one-half wave length of the comparison frequency. Along the base line lanes are separated by about 1,500 feet. A Decca reading is composed of three parts: The Zone letter indicating the zone location in the area; the lane number, and the decimal subdivision equivalent to the phase reading. A typical reading might be E 13.18.

Like all hyperbolic systems the linear separation of the lines of position increases with distance from the stations. At six baseline lengths distant from the middle of the base line the lines are twelve times farther apart than along the base line. With a 50 mile base line the distance difference distinguishable along the base line is 15 feet, but at 300 miles it is 180 feet. However,

at a range of 300 miles the hyperbolic lines cross at an angle of 10 degrees which will give a parallelogram of fix 360 feet wide and 3,600 feet long in the direction of the lines of position.

Lane identification is a problem which plagues all continuous wave hyperbolic systems. Since phase difference is the quantity measured, all lanes look alike and a receiver could be in anyone of a number of locations which would give the same phase difference rating. The method usually employed for lane identification is to set the receiver dial readings to the proper values for a known point when the ship is at that point (such as a moored buoy, end of a pier, etc.). Then as the ship later moves about the area the dials will automatically record the proper zone and lane changes as well as the decimal subdivisions determined by the phase difference. If, however, the signal is lost due to interference, breakdown, sky wave effect, etc. and the ship has moved during the period of interrupted reception, it will be necessary to return to the calibration point to reset the dial readings to the correct values.

Decca signals have been received out to distance of 1,000 miles but in order to remain within an areas of reasonable intersection of the hyperbolic lines (15 degrees or greater) a range of 200 nautical miles for a 50 mile base line is about the most than can be expected. Little sky wave interference is observed at night out to 100 miles but beyond that distance the results become increasingly less accurate until at 240 miles the system breaks down due to sky wave effects.

The operation of the Decca system is illustrated in Figure

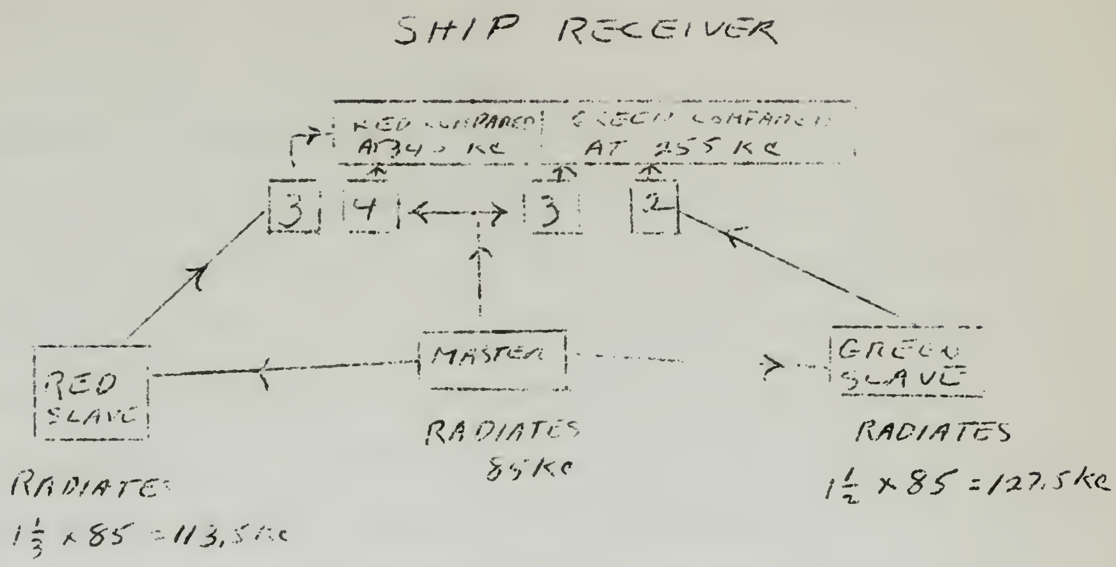


FIGURE 26: Decca System

26. The master station radiates a continuous wave signal of 85 KC which is received by the receivers of the ship and the two slave stations. These receivers multiply the frequencies by the factors shown on the sketch. The red slave station transmits to the ship at the increased frequency of 113.5 KC and the green slave transmits at 127.5 KC. These frequencies when received at the ship are magnified by the appropriate factor so that the signal from the red slave will be increased to 340 KC and the green signal will be increased to 255 KC. Since the signal transmitted directly from the master station to the ship had been split and increased to 340 KC and 255 KC it is now possible to compare the phases of the comparison frequencies. The phase difference of the signal from the red slave and the master will be measured at the common comparison frequency of 340 KC and the phase difference for the green pair will be measured at the common comparison frequency of 255 KC.

Decca is used extensively for ship navigation in the English Channel and off the Scandinavian Peninsula. However, it has even more use as a precision method for control of hydrographic surveys. It has been thoroughly tested for errors and found to have a standard error of ± 25 feet. On this basis it has been used successfully by the British, Finns, Norwegians, Swedes, and many other foreign governments for surveying considerable areas of the ocean. Of the precise long range electronic systems used for hydrographic surveys, Decca is the only one that has been thoroughly tested; it is not expensive; and is on a production basis so that delivery can easily be made within six months of receipt of a contract order.

SPECIFIC ELECTRONIC POSITIONING SYSTEMS

D. LORAC

Lorac, standing for LOng Range ACcuracy, is manufactured by the Seismograph Service Corporation, P. O. Box 1590, Kennedy Building, Tulsa 1, Oklahoma. It is a phase comparison continuous wave hyperbolic system based on the principle of measuring the phase difference between the signals emitted by two continuous wave shore transmitters. A mobile receive carried aboard the survey vessel produces a phase difference meter reading which determines the position of the receiver with respect to a family of hyperbolas. A second pair of stations is so situated that its hyperbolas will intersect those of the first pair, forming a grid. The central station is common to two pairs so a total of only three stations is needed, designated Red Slave, Master, and Green Slave.

Lorac solves the problem of synchronization, or phase locking of the transmitters by a unique system of transmitting a modulated "reference" signal for phase comparison purposes. By having each slave station act as reference station for the other, synchronization is obtained (as we shall see later) without the use of any additional monitor stations. Instead of regular continuous wave phase angles, the phase of a heterodyne beat note in the audio range is used which, according to the manufacturer can be measured to an accuracy of 3.6 degrees, corresponding to a movement of ± 2.5 feet along the base line. This is arrived at by the following reasoning: For frequencies on the order of 2,000 KC the wave length is

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

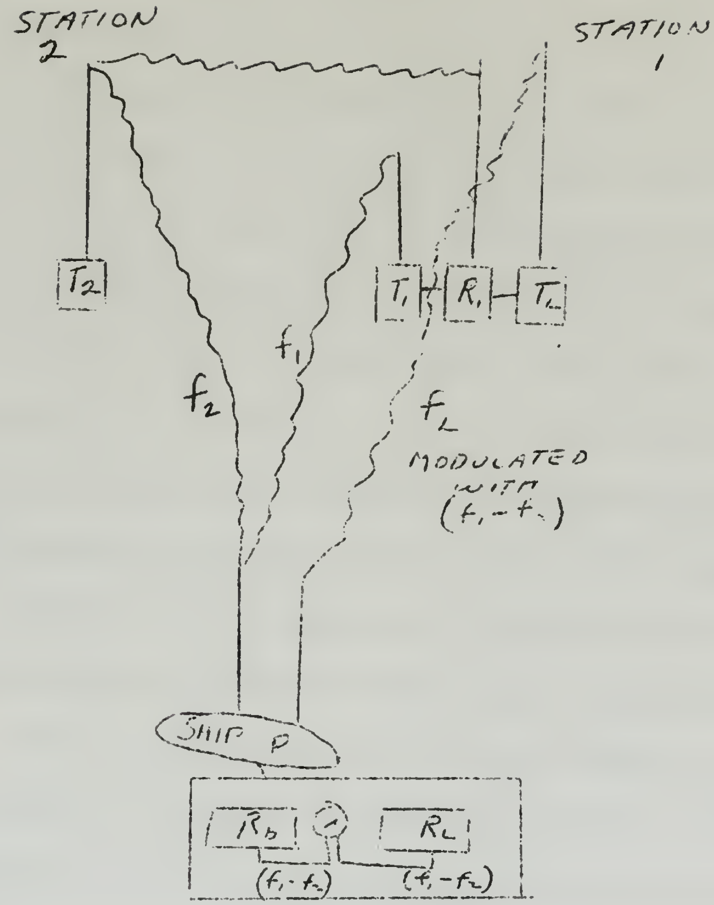
[illegible]

$$186{,}000\ \frac{\text{mi}}{\text{sec}} \times \frac{1}{2{,}000{,}000\ \frac{\text{cyc}}{\text{sec}}} \times 5{,}280\ \frac{\text{ft.}}{\text{mi}} = 500\text{ ft.}$$

On the base line between transmitters, one-half wave length is one lane and is approximately 250 feet in length, constituting a phase shift of 360 degrees. Since this can be read with an accuracy of $\pm$ 3.6 degrees, an instrumental accuracy (excluding propagation effects) of 1/100th of 250 feet, or $\pm$ 2.5 feet, is reasonable to expect. This will be degraded in accuracy as the ship moves away from the base line, in accordance with the geometrical effects for a hyperbolic system.

The method of accomplishing the measurement of phase difference is relatively simple in practice but is difficult to explain with words. Careful attention should be paid to Figures 27 and 28, and each step carefully followed. To begin with the basic system, Figure 27:

Transmitters, T_1 and T_2, located at shore positions 1 and 2, emit continuous wave signals at frequencies f_1 and f_2 respectively. The difference of these frequencies, $(f_1 - f_2)$, is an audio frequency. The signals are received at the ship on receiver R_b and at shore position 1 on receiver R_1, heterodyning in both receivers to produce the audio beat frequency $(f_1 - f_2)$. The beat of receiver R_1, $(f_1 - f_2)$ is used to modulate transmitter T_2 operating at a frequency f_2. Receiver R_2 on the ship demodulates the signal f_2 and obtains the beat frequency $(f_1 - f_2)$ also called the reference beat note. This reference beat note is phase compared with the audio beat frequency of the ship's receiver R_b to give the required phase difference reading.



Transmitters T_1 and T_2 are continuous wave transmitters and T_2 is on amplitude-modulated transmitter.

FIGURE 27: Lorac Basic System

Each of the two beat frequencies arrives at the ship with a phase value governed by the difference in the distances the original carriers have traveled before reaching the heterodyning receivers to form the best notes. The best note, transferred to the ship as a modulated carrier, has a phase value which is constant regardless of the position of the ship. Because of this constant phase value it is called a "reference beat note". The heterodyne beat note received on the ship's receiver R_B directly from transmitters T_1 and T_2 has a phase value which is dependent on the relative distances between each of the shore transmitting antennas and the ship receiving antenna. If slight electrical changes take place at either shore transmitter resulting in variations in phase, both beat notes will vary an equal amount so that the effects will be cancelled and the phase meter reading will not change. This important feature completely obviates phase synchronization problems.

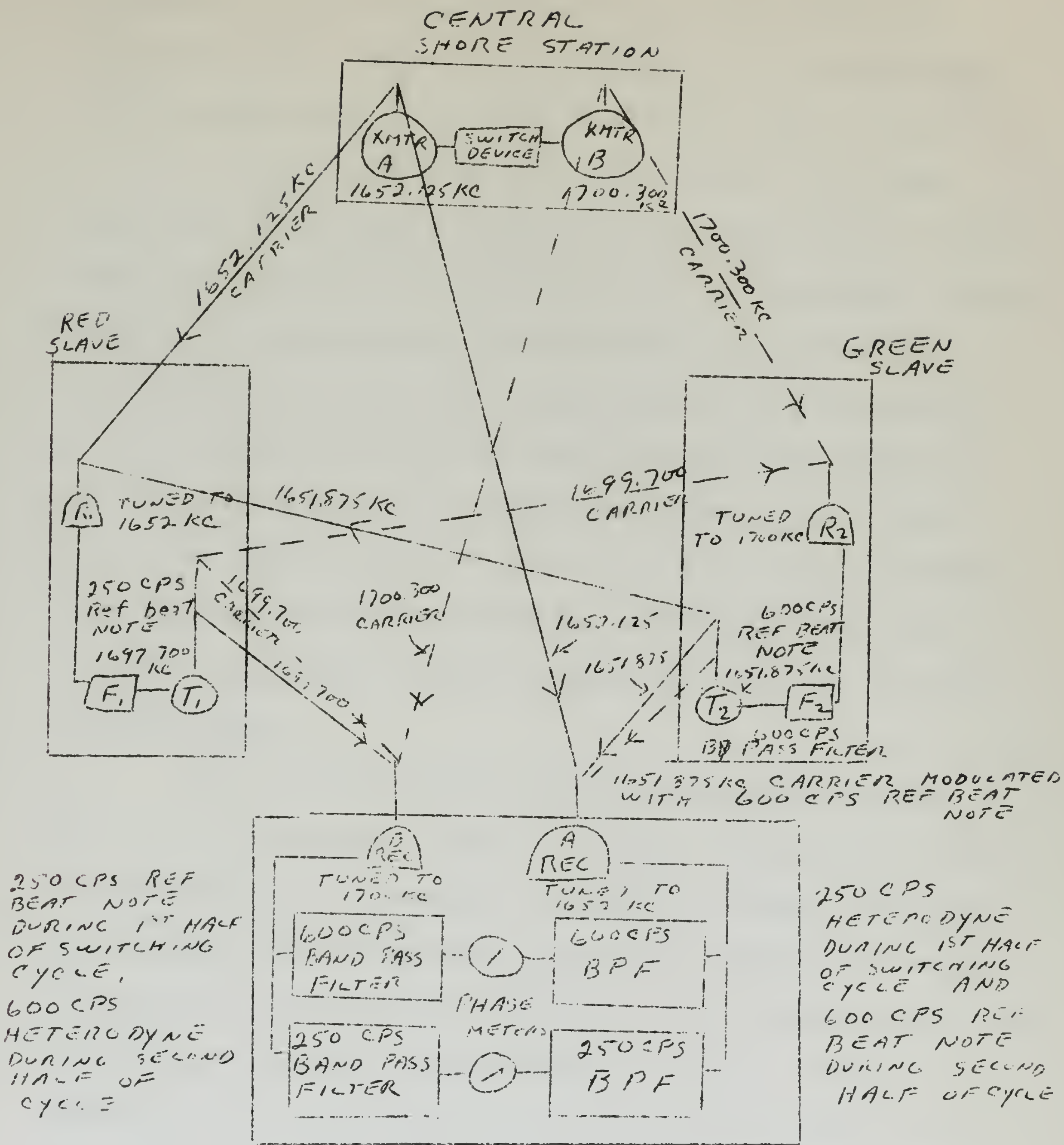
When the ship moves so there is a change in its relative distance from the two shore transmitters the phase value of the $(f_1 - f_2)$ beat note at receiver R_b will change but the phase value of the reference beat note will not change. Consequently the phase meter will record the change in phase relationship between the two beat notes.

If the ship continues to move relative to the two transmitters the phase will change until a complete 360 degree phase change has taken place. At that time an integrating counter mechanism in the phase meter adds or subtracts a digit to indicate that the ship has entered a new lane. The phase meter continues to indicate the

position within the new lane. If the moving ship is steered so there is no change in the reading of the phase meter the course followed will be along one of the hyperbolas of the coordinate system.

The entire instrumental system is completely duplicated to give two families of hyperbolas which intersect each other. This is for the basic design. The actual production model , however, contains many improvements while still following the same basic principles. By providing a switching device at the central shore station (master station) the independent reference transmitters are eliminated, fewer receivers are required aboard ship, and the extra reference transmitter frequency channels are not required. The system, therefore, functions on two frequency channels and requires only four transmitters and two receivers to establish a hyperbolic grid. A pair of receivers and a pair of phase meters is all that is required for each sounding vessel.

Figure 28 illustrates the operation of Lorac Type A system. Transmitters A and B at the central shore station are alternately switched on and off by a suitable switching device. The solid lines depict the significant radiations during the first half of the switching cycle when transmitter A is transmitting. The dashed lines show the significant emissions during the second half of the switching cycle when B is transmitting. The second half of the cycle will be easiest to follow since it corresponds in general configuration with the basic system, (Figure 27). Transmitter T_2 serves as the heterodyning complement of transmitter A during the first half of the cycle and as a reference transmitter to convey the reference beat



SOLID LINES INDICATE RADIATIONS DURING FIRST HALF OF SWITCHING CYCLE

DOTTED LINES ARE SECOND HALF OF SWITCHING CYCLE

FIGURE 28: Lorac Type A System

frequency of the other transmitter pair to the ship during the second half cycle. Transmitter T_1 performs the same functions for the other half of the system but in the reverse cyclical order.

In common with other continuous wave phase comparison systems, the Lorac system is a differential distance measuring system, absolute distances to the transmitters being give indirectly. Before undertaking to navigate with this system the phase meters must be set to indicate the proper phase relationships at a known location. The phase meters will assume the correct phase position within the lane (decimal values) but the correct lane number must be cranked in the phase meters. If it is desired to operate daily on a 24 hour basis the Lorac end stations should not be separated by more than 55 miles. Beyond this separation distance the effects of sky waves introduce errors.

Lorac equipment has been used with excellent results by the U. S. Navy Hydrographic Office for precise channel surveys in the Persian Gulf and for hydrographic surveys in the Bahamas, and by the Bell Telephone Company for cable laying in the Caribbean Sea for the U. S. Air Force guided missile test range. It is unfortunate that the system has not been fully tested as Decca and Shoran. However, experience of survey ships operating in the Persian Gulf under the control of the U. S. Navy Hydrographic Office shows that the relative accuracy of positioning is better than the possible plotting accuracy. A ship channel survey about 50 miles from the shore stations was completed successfully with sounding lines spaced 50 yards apart on a scale of 1:50,000. Slight changes of course by the survey ship were

immediately detected by the Lorac controlled plot. This system will be a very useful method for precise work as soon as it is fully tested to discover the errors involved in the system.

RAYDIST

Raydist, is manufactured by the Hastings Instrument Company, Inc. of Hampton, Virginia. It is a phase comparison continuous wave hyperbolic system that is extremely flexible and can be set up to give many different arrangements:

a) Hyperbolic ranging with automatic recording on shore or on the mobile station.

b) Hyperbolic positioning based on phase difference between signals emitted by mobile receiver and 3 shore stations.

c) Hyperbolic positioning based on phase difference between signals emitted by two or more pairs of shore stations. This system permits an unlimited number of users and is employed extensively for off shore oil exploration.

d) Circular ranging systems.

In any of the above set ups arrangements can be made for automatic recording of positioning on the mobile receiver, at one of the shore stations, or on both the mobile receiver and at a shore station. This remarkable flexibility and adaptation to different requirements makes Raydist an extremely valuable system for a variety of purposes. It is used for acceptance trials of new construction ships in determining speed and maneuvering characteristics of the ships; for tracking guided missiles; for cable laying; for off shore oil exploration; and for hydrographic surveying.

Raydist solves the problem of phase locking by use of a reference station permanently located in relation to the three fixed main shore transmitting stations. An understanding of the principle of operation can be had from an inspection of the schematic diagram (Figure 29). The transmitter aboard ship (mobile receiver) transmits a continuous wave signal at 2.068 mc which is received at the Red and Green slave stations and the master stations, (designated the Yellow station by the Manufacturer). A reference transmitter located at a known fixed position with relation to the shore stations also transmits a continuous wave signal at 2.0072 mc which is picked up by the three shore stations. The receiver at the Red Slave receives the signal from the ship and from the reference station and introduces the resultant signal to the Red transmitter. This signal has a definite phase depending on the distance difference of ship to Red slave and reference stations to Red Slave. The signals is modulated and transmitted from the Red Slave to the ship at 29.720 mc. Similarly the Yellow receiver handles the signals from the reference station and the ship, introduces the resultant signal to the Yellow transmitter which transmits the modulated signal to the ship at 29.760 mc. The phase difference between the signals transmitted by the Red and Yellow stations locates the ship on a Red hyperbolic curve. Similarly the phase difference between Green and Yellow signals locates the ship on a Green hyperbolic curve. The intersection of the two curves

determines the ships position. It can be seen that any error of transmission from the ship or the reference transmitters will be reflected in the phase transmitted by each shore station so that while the absolute values will be in error, the phase difference will be correct. This insures phase locking of the system.

The description above (Figure 29) is a general description one one of the basic systems. However, by locating the ship's transmitter at a reference relay station the transmission by the mobile receiver is eliminated and an unlimited number of users can work on the same network. Each of the transmitters can be made to differ in frequency from the others in the system by an audio note. Radio frequency phase measurements are reduced to audio frequency phase measurement by the heterodyne signals between the transmitters, greatly simplifying the problem of comparing the phase of the received signals. The transmittal of audio beats between the transmitters results in audio signals being received at the fixed relay station and in the ship. The beat notes between these signals received at the relay station modulate the relay transmitter which in turn broadcasts to all ships using the system. The audio notes received from the relay stations are compared in phase with the audio beat notes received directly from the shore stations. The phase difference locates the ship on a hyperbolic line of position.

As with other phase comparison systems, the phase meters must be set to indicate the proper phase relationship, at a known location. The problem of lane identification can be resolved by use of additional frequencies or use of additional shore stations, but either systems results in such an unwieldy and cumbersome system that the navigator can become confused with excessive number of lines crossing each other.

Raydist has been used successfully for hydrographic surveys by the Portuguesse Navy and by the Norfolk District of the U. S. Army Corps of Engineers; for ship acceptance trials in the speed trials of the S. S. United States; and for missile tracking and cable laying for the Downrange Survey for the U. S. Air Force. It is a marvellously flexible system with a variety of applications. It also has not been adequately tested but the little test results available indicate that it has an accuracy of at least $\pm$ 50 feet within the area of operation out to a range of about 100 miles.

TABLE III

COMPARISON OF VARIOUS POSITIONING SYSTEMS

Explanation: This table was originally designed to compare on one sheet the various characteristics of the different systems discussed in this report. However, such a table cannot be reproduced on a normal-carriage typewriter. In the absence of any other means of reproducing the table, the pertinent data for each system is published on individual pages. For the most part the table entries are self-explanatory but note should be taken of the following.

Observed error, or error claimed by manufacturer: Observed error is used wherever a system has been adequately tested. However, since some of the electronic systems have not been adequately tested the manufacturer's claimed accuracy for those systems is used. These error values should be compared with the theoretical values taken from Figures 15, 16, and 20.

Portability: This is a very important factor for survey groups operating in remote areas away from roads, pier space, or other transportation facilities. Therefore, in any system the LARGEST component of any shore station should be small enough to be carried by a jeep or by an HRS type helicopter. Needless to say, it

[illegible]

should also be able to be offloaded by small boat, and light enough to be easily handled by a six to eight man working party. For boat work the equipment required for positioning a sound boat should be small and rugged enough to be used in the boat.

<u>Cost</u>: The following hypothetical assumptions were made in arriving at the relative cost:

1. The area to be surveyed has 50 miles of coastline and is to be surveyed as far to seaward as the positioning system permits.
2. The survey group will be composed of three ships to do the offshore hydrography and four sound boats for the inshore work. For such a group a total of seven receivers for electronic surveying or twenty-one sextants for visual surveying will be required.
3. For visual sextant control the spacing of main scheme triangulation is a tripod signal every four miles along the coast and four miles inland for a total of twenty-four tripod signals. The spacing of secondary signals is every 3/4 mile along the coast for a total of sixty centerpole signals. Electronic surveying requires a main scheme triangulation net with

(Table III)

(COMPARISON OF VARIOUS POSITIONING SYSTEMS)

a tripod signal about every twelve miles for a total of eight tripod signals.

4. The cost of various basic items: $450.00 for each sextant, $30.00 for each tripod, and $20.00 for each centerpole.

(Table III)

(COMPARISON OF VARIOUS POSITIONING SYSTEMS)

CELESTIAL OBSERVATION

Maximum range: Good all over the earth's surface.

Observed error: m = ± 0'.2 (± 1.200 feet), a = ±0'.66 (3,960 feet).

Number of simultaneous users: Unlimited.

Portability: For precise work the only equipment needed is

1) Sextant with endless tangent screw.
2) Break circuit chronometer and recording chronograph checked by radio time signals.
3) Emphermis or specially prepared almanac.
4) Special large scale charts for plotting results.

Advantages:

1) Can determine position anywhere in the world independent of special electronic equipment.
2) Unlimited number of users.

Disadvantages:

1) Cannot be used during darkness or decreased visibility.
2) Unsuitable for precise surveying because of the unknown errors involved.
3) Only gives two fixes per day (weather permitting).

(Table III)

(COMPARISON OF VARIOUS POSITIONING SYSTEMS)

SEXTANT THREE-POINT POSITIONING

Maximum range: 10 miles from coast.

Observed error: ± 10 yards at scale of 1:10,000 and proportionately greater for smaller scales.

Number of simultaneous users: Unlimited.

Portability: Only equipment required is two sextants, three-arm protractor, and regular plotting chart.

Installation time: About one week to establish first quadrilateral of triangulation net and to cut in secondary signals for use by sounding craft.

Maintenance personnel: None.

Cost: For a 7 sounding unit survey group:

21 sextants	$ 9,400.00
7 protractors	700.00
triangulation signals	2,000.00
	$12,100.00

Advantages: Quick, simple, easily understood method providing adequate accuracy for routine hydrographic surveying.

Disadvantages:
1) Limited range.
2) Restricted to periods of good visibility.
3) Not accurate enough for precise cable laying, guided missile, or mine laying work.

(Table III)

(COMPARISON OF VARIOUS POSITIONING SYSTEMS)

SHORAN

Basic Class: Circular.

Frequency bond: 225 - 330 MC.

Type of Transmission: Pulse.

Method of time measurement: Distance between pulses.

Maximum range: Line of sight, approximately 20 nautical miles.

Observed error: m = $\pm$ 24 ft., σ = $\pm$ 79 ft. at 100 miles.

Minimum average error of fix (Figure 15): $\pm$ 50 ft.

Average error of fix as function of distance (Figure 16) for

D = baseline	:	$\pm$ 55 ft.
D = 2x baseline	:	$\pm$ 85 ft.
D = 3x baseline	:	$\pm$ 170 ft.

Number of simultaneous users: Limited to about 3 with a fix every 3 minutes.

Portability: Shore stations weigh about 1,6000 pounds per station. The bulkiest component is a mast box measuring 132" x 20" x 11" and weighing 27½ pounds. The heaviest item is the transmitter weighing 230 pounds and measuring 38½" x 25" x 20". The equipment required for use aboard ship amounts to only 300 pounds.

Installation: 2 days per station.

Maintenance personnel after system is in operation: 2 men at each shore station.

Cost: $55,000.00 for complete system including 3 shipboard units.

Advantages:
1) Extremely accurate.
2) Unhampered by visibility or darkness.
3) Easily constructed plotting charts.
4) Small geometrical effects.
5) Easy installation.
6) No disruption of operations due to skywaves.

(Table III)

(COMPARISON OF VARIOUS POSITIONIN SYSTEMS)

(SHORAN)

Disadvantages:

1) Restricted to line of sight ranges.
2) Limited number of users requiring time sharing schedule if more than one user is in the area.
3) Fairly elaborate shipboard equipment.

(Table III)

(COMPARISON OF VARIOUS POSITIONING SYSTEMS)

EPI

Basic class: Circular.

Frequency band: 1,850 kc.

Type of transmission: Pulse.

Method of time measurement: Equalized and superimposed pulses.

Maximum range: 480 nautical miles.

Observed error: m = $\pm$ 150 ft., a = $\pm$ 495 ft. at 400 miles.

Minimum average error of fix (Figure 15): $\pm$ 500 ft.

Average error of fix as function of distance (Figure 16) for

D = baseline	:	$\pm$ 625 ft.
D = 2 x baseline	:	$\pm$ 1,100 ft.
D = 3 x baseline	:	$\pm$ 1,550 ft.

Number of simultaneous users: Limited to about 3 with a fix every 3 minutes.

Portability: The largest component of the shore installation weighs 200 pounds and measures 24" x 24" x 33". Antenna masts are 10 feet long measuring 12" x 12" and weighing 16 pounds each.

Installation time: 6 days per station.

Maintenance personnel: 4 to 6 men at each station.

Cost: Not yet in production for general use except for U. S. C. and G. S. vessels.

Advantages:

1) Ranges out to 480 nautical miles.
2) High accuracy.
3) Unhampered by visibility or darkness.
4) Easily constructed plotting charts.
5) No disruption of operation, due to sky waves.
6) Small geometrical effects.

(Table III)

(COMPARISON OF VARIOUS POSITIONING SYSTEMS)

(EPI)

Disadvantages:

1) Limited number of users.
2) Not yet in general production.
3) Fairly elaborate shipboard equipment.
4) Minimum range of 14 miles.
5) Must be frequently calibrated against a known distance.
6) Not adaptable to small boats because of power requirements.
7) Before each fix and at time of fix pulse must be synchronized at the ground stations by making coincidence over the entire length of the pulse. This requires a constant watch be maintained.

(Table III)

(COMPARISON OF VARIOUS POSITIONING SYSTEMS)

LORAN

Basic class: Hyperbolic.

Frequency band: 1750-1950 kc.

Type of transmission: Pulse.

Method of measurement: Equalized and superimposed pulses.

Maximum range: 750 nautical miles using ground waves, 1,400 nautical miles using sky waves.

Observed error: $m = \pm$ 0.2% of distance.
$a = \pm$ 0.66% of distance.

Minimum average error of fix (Figure 15): $\pm$ 2,600 ft.

Average error of fix as function of distance (Figure 20) for

D = baseline	:	$\pm$ 1.25 nautical miles.
D = 2x baseline	:	$\pm$ 2.0 nautical miles.
D = 3x baseline	:	$\pm$ 3.2 nautical miles.

Number of simultaneous users: Unlimited.

Portability: Heavy permanent shore installations beyond the capacity of survey ships. Shipboard receiver weighs about 100 pounds.

Cost: Approximately $2,000.00 per shipboard receiver; cost of shore installations borne by various governments.

Advantages: Provides continuous positioning for very long ranges at a small cost for an unlimited number of users.

Disadvantages: Not suitable for precise hydrographic surveying because of lack of accuracy. However, in its area of coverage it is vastly superior to celestial observations because it is independent of time or weather conditions. It is subject to interference from local radio transmissions.

(Table III)

(COMPARISON OF VARIOUS POSITIONING SYSTEMS)

DECCA

Basic class: Hyperbolic.

Frequency band: 85 kc - 127.5 kc.

Type of transmission: Continuous wave.

Method of time measurement: Phase comparison.

Maximum range: 150 nautical miles without appreciable sky wave and interference, out to 200 miles for daylight operations.

Observed error: m = ± 25 ft., a = ± 82.5 ft. at 50 miles.

Minimum average error of fix (Figure 15): ± 50 ft.

Average error of fix as function of distance (Figure 20) for

D = baseline	:	± 150 ft.
D = 2x baseline	:	± 375 ft.
D = 3x baseline	:	± 800 ft.

Number of simultaneous users: Unlimited.

Portability: Each shore station weighs about 1,500 pounds and can be divided into components small enough to be portable by helicopter. Shipboard receiver weighs220 pounds.

Installation time: About 5 days per station to erect and put in operation

Maintenance personnel: 4 men for continuous operation.

Cost: $100,000.00 for complete system and one receiver. Each additional receiver costs $2,850.

Advantages:

1) High accuracy,
2) Ranges to 150 nautical miles at night and out to 200 nautical miles during daylight,
3) Unhampered by visibility or darkness.
4) Unlimited number of users.
5) Only precise continuous wave system that has been fully tested for error analysis.

(Table III)

(COMPARISON OF VARIOUS POSITIONING SYSTEMS)

(DECCA)

Disadvantages:

1) Requires precomputed hyperbolic grid.
2) Any interruption of signal causes loss of lane count requiring return to a known calibration point.
3) Requires use of monitor station to insure phase locking.
4) If desired to change one of the frequencies by other than a multiple of the basic frequency, then the other frequencies must also be changed because all frequencies are critically related.

(Table III)

(COMPARISON OF VARIOUS POSITIONING SYSTEMS)

LORAC

Basic class: Hyperbolic.

Frequency band: 1,772 kc to 1,797 kc. These can be easily changed by substitution of other frequency crystals.

Type of transmission: Continuous wave.

Method of time measurement: Phase comparison.

Maximum range: 35 miles without appreciable sky wave interference; out to 200 miles for daylight operation.

Manufacturer's stated error: Untested for errors but manufacturer claims ± 2.5 feet along baseline.

Minimum average error of fix (Figure 15): ± 2.5 feet.

Average error of fix as function of distance (Figure 20) for

D ≈	baseline	:	± 7.5	ft.
D ≈	2x baseline	:	± 18	ft.
D ≈	3x baseline	:	± 40	ft.

Number of simultaneous users: Unlimited.

Portability: The 100 watt set shore stations weighing about 1,500 pounds each, with the largest single component weighing 190 pounds and measuring 19" x 25" x 37". The 500 watt set is considerably larger and is not as portable. For the 500 watt set all pieces of equipment are portable by helicopter but the power supply is a bulky item ranging in weight from 800 to 1,700 pounds depending on the type used.

Installation time: About five days per station to erect and put in operation.

Maintenance personnel: 4 men per station for continuous 24 hour operation.

Cost: Complete shore installation without spare parts and without power plants is $50,000.00. Each receiver costs $11,000.

(Table III)

(COMPARISON OF VARIOUS POSITIONING SYSTEMS)

(LORAC)

Advantages:

1) Results of field tests are not available from the manufacturer for publication but THEORETICALLY this is one of the most accurate of all positioning systems.
2) Ranges to 35 nautical miles for continuous operation without sky wave effects, out to 200 miles for daytime operations.
3) Unhampered by visibility or darkness.
4) Unlimited number of users.
5) Does not require monitor or reference receiver for phase locking.

Disadvantages:

1) Requires precomputed hyperbolic grid.
2) Any interruption of signals causes loss of lane count requiring return to a known calibration point.
3) Only available in limited quantities since a full scale production model is not yet ready for distribution.

(Table III)

RAYDIST

Basic Class: Hyperbolic.

Frequency band: Various frequencies can be used depending on the bands available.

Type of transmission: Continuous wave.

Method of time measurement: Phase comparison.

Maximum range: Out to 200 miles, depending on frequencies used.

Observed error: Results of few test indicate an accuracy of ± 50 ft.

Number of simultaneous users: Depending on system that is used, but can be set up for an unlimited number of users.

Portability: Portable within the limits of a hydrographic survey group. Detailed weight and cube specifications were not furnished by manufacturer.

Installation time: About five days per station to erect and put in operation.

Maintenance personnel: Four men per station for continuous 24-hour operation.

Cost: $50,000.

Advantages:
1) High accuracy.
2) Long ranges.
3) Unhampered by visibility or darkness
4) Unlimited number of users.
5) Extremely flexible and adaptable to a variety of survey requirements,
6) Is in a production status.

Disadvantages:
1) Hyperbolic set up requires precomputed grid.
2) Any interrruption of signals causes loss of lane count requiring return to a known calibration point.

CONCLUSIONS AND RECOMMENDATIONS

On the basis of the data contained in this report, especially Table III, I have arrived at the following conclusions pertaining to positioning methods to be used in precise hydrographic surveys:

1. Celestial observations at sea with a marine sextant should be used only where more precise control is not available. This method is the least precise of all methods discussed.

2. Loran is almost as accurate as celestial observations, but has the decided advantage that it is independent of time, is unaffected by weather, and provides continuous positioning in the area of its coverage.

3. Sextant three-point positioning is accurate within the plotting requirements of present day hydrographic surveys, excluding cable surveys or other precise operations. However, it has such a limited range that it will provide hydrographic coverage only for the smallest of coastal vessels. Modern navigators stay as far off the coast as possible so any method that does not furnish soundings in areas required by the users is better off replaced by a method that will. It is extremely inefficient, time consuming, and expensive to carry out a survey with two methods of control: visual sextant control out to the limits of visibility and electronic methods from there seaward. Therefore, it is recommended that sextant three-point positioning be used only for surveying small bays or lakes. The method may also have

some value for quick "spot" surveys of two or three weeks duration in Artic areas where daylight is continuous and where transient electronic interference may be expected.

4. Shoran for inshore control and EPI for offshore hydrography is highly recommended for survey vessels operating singly or in small groups. The time sharing required in using these remarkable systems make them unsuitable for large surveying groups. However, for individual ships there is no better combination.

5. For large surveying expeditions any of the three continuous wave hyperbolic systems should be used depending on the personal preferences of the agency carrying out the survey. These systems provide accurate long range positioning for an unlimited number of users. Since at least 50 miles of triangulation must be completed before the hyperbolic curves can be computed for the proposed sites of the three shore stations it is recommended:

a) that advance party be sent to obtain the necessary triangulation data so that the hyperbolic curves will be ready upon arrival in the survey area. Or

b) that the first part of the survey be carried out using Shoran control. A low order triangulation net, accurate enough for preliminary work, can be quickly established in a few weeks and the curves can then be constructed graphically aboard ship. Sounding control using the hyperbolic system can be started as soon as the curves have been constructed. As the

season progresses the triangulartion can be re-observed more precisely and then accurate curves produced by high speed computing machines for the final smooth plot work. Or

c) that after thorough reconnaissance upon arrival in the area the three stations be established at the most suitable locations in the area. Then determine the length of the baselines between the stations by the method of crossing the base line extensions to record the maximum and minimum readings. Subtracting the minimum from the maximum readings will give the number of lanes in the base line from which the base line length can be readily computed. The angle of intersection of the base lines and the azimuths can be picked directly from the photographs or maps of the area or by observing rockets shot off from each station at night. This gives enough data to construct the hyperbolic curves graphically and then run sound lines. As the season progresses accurate triangulation will be established and precise curves can then be computed. This method is extremely rough but will be good enough to get the survey started and for preliminary boat sheet work that can be corrected in the smooth plot. It is only an expedient to be attempted if a) or b)

above can not be used.

The advantages and disadvantages of the various systems have been described in Table III and recommendations concerning each system have been made in the previous paragraphs. The superiority of electronic positioning systems is obvious but there is still much room for improvement. The problems of removing sky wave effects and of obtaining an absolutely correct value for the velocity of radio wave progogation under all conditions of weather and terrain appear almost insurmountable but must be solved if electronic methods are ever going to provide precise first-order WORLD WIDE coverage. Every approach so far has met with failure until it appears to many that electronic methods have reached their peak of efficiency and can no longer be improved. The situation is analogous to a young baby learning to walk. The baby will crawl over to a piece of furniture and laboriously pull himself erect only to fall down when he tries to take his first step. He will crawl over to the nearest hand-hold and carry out the same procedure time after time until he finally succeeds. Now suppose after a few falls the baby should sit back to analyse his procedure in the approved method and should conclude: " I can see that the result of this endeavor will always be that I will fall flat on my haunches. This indicates I have reached the peak of my ability in locomotion. Ergo, I had best be content with crawling." The question that enters my mind at this point is: If the above train of thought had ever entered the heads of infants, how could

the human race ever get beyond the crawling stage? We must continue our search for new methods, new techniques, and new instruments, while constantly striving to improve those presently in use.

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